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Adaptive Generalized Synchronization of Drive-Response Neural Networks with Time-Varying Delay

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Abstract

This paper studies the generalized synchronization of a class of drive-response neural networks with time-varying delay. When the topological structures of the drive-response neural networks are known, by designing an appropriate nonlinear adaptive controller, the generalized synchronization of these two networks is obtained based on Lyapunov stability theory and LaSalle's invariance principle.

Keywords

Generalized Synchronization, Drive-Response Neural Network, Time-Varying Delay, Adaptive Controller

1. Introduction

There are various complex networks in nature and human society, such as transportation network, biological network, social relationship network, neural network and so on [1] [2] [3]. In recent years, complex networks have attracted the attention of scholars in many fields including biology, engineering, economics, neuroscience, mathematics, physics.

They have been become a research hotspot in academic circles, many interesting and valuable results are obtained [4] [5] [6] [7]. Many studies have been devoted to the synchronization of complex dynamic networks due to its wide application in the real world [8] [9] [10] [11]. Synchronization is a collective phenomenon and behavior, its principle can be used to promote social production and human activities, for instance the security of communication, the development of laser equipment and nuclear magnetic resonance instrument [12].

Therefore, the research on synchronization of complex dynamic networks has very important practical significance.

The consistency problem of all nodes in a complex network is internal synchronization. Various synchronization methods have been proposed, such as adaptive control [13] [14] [15], impulse control [16] [17], pinning control [18] [19]. In fact, synchronization can also be realized between two networks, that is, external synchronization, including complete synchronization [20], projective synchronization [21] [22], generalized synchronization [23] [24] [25]. The projective synchronization of two complex networks with the same node dynamics is studied in the literature [22]. By designing a nonlinear controller, the generalized synchronization of two complex networks with different node dynamics was realized, but the time delay was not considered [24]. In [25], the generalized synchronization for two complex networks with time-varying delay coupling was investigated.

Neural networks, as a special kind of complex networks, have been received considerable attention because of its potential application in neurophysiology, automatic control, image processing. The synchronization of two neural networks with time delay was studied in the literature [26]. The projective synchronization of neural networks without time delay was obtained in [21]. The literature [27] realized the projective synchronization of a class of neural networks by using adaptive feedback control method.

However, there are few studies on the generalized synchronization of a class of neural networks with time-varying delay. This paper is concerned with the problem of generalized synchronization for drive-response neural networks with time-varying delay. If the topological structures of the systems are known, by designing an appropriate nonlinear adaptive controller, the generalized synchronization between these two networks can be achieved based on Lyapunov stability theory and LaSalle's invariance principle.

This paper is organized as follows. In Section 2, model description and some important preliminaries are given. In Section 3, the main result is presented.

2. Model and Preliminaries

Consider a neural network with time-varying delay, which can be described by the following equation:

$$\begin{aligned}\dot{x}_i(t) = &-Cx_i(t) + Af\left(x_i(t)\right) + Bf\left(x_i\left(t-\tau(t)\right)\right) \\ &+ c_0\sum_{j=1}^{N} d_{ij}\Gamma_1 x_j(t) \quad (i = 1, 2, \cdots, N)\end{aligned} \tag{1}$$

$x_i(t) = \left(x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)\right)^{\mathrm{T}} \in R^n$ is the state vector of the i-th node at time t; $f: R^n \to R^n$ is a smooth vector function; $\tau(t)$ is the time varying delay; $C = diag\left(c_1, c_2, \cdots, c_n\right) \in R^{n\times n}$ with $c_i > 0\left(i = 1, 2, \cdots, n\right)$ denotes the state feedback coefficient matrix; $A = \left(a_{ij}\right) \in R^{n\times n}$ and $B = \left(b_{ij}\right) \in R^{n\times n}$ are respectively the weight and delayed weight matrices; $c_0 \in R^{+}$ is the network coupling strength; external coupling configuration matrix $D=\left(d_{ij}\right)\in R^{N\times N}$ represents the network topology and the coupling strength between nodes, the following conditions are met: if there is a connection between node i and node j, then $d_{ij}>0$, if not $d_{ij}=0\left(i\neq j\right)$, and the diagonal element $d_{ii}=-\sum_{j=1,j\neq i}^{N}d_{ij}$, $i=1,2,\cdots,N$; matrix $\Gamma_1\in R^{n\times n}$ is a known constant positive definite diagonal matrix, which represents the internal coupling of the network.

We regard network (1) as a drive network, and the following equation is the corresponding response network:

$$\begin{aligned}\dot{y}_i\left(t\right)=&-Cy_i\left(t\right)+Ag\left(y_i\left(t\right)\right)+Bg\left(y_i\left(t-\tau\left(t\right)\right)\right)\\&+c_0\sum_{j=1}^{N}g_{ij}\Gamma_2y_j\left(t\right)+u_i\quad\left(i=1,2,\cdots,N\right)\end{aligned}\tag{2}$$

$y_i\left(t\right)=\left(y_{i1}\left(t\right),y_{i2}\left(t\right),\cdots,y_{in}\left(t\right)\right)^{\mathrm{T}}\in R^n$ is the state vector of the i-th node at time t; $g:R^n\to R^n$ is a smooth vector function; $G=\left(g_{ij}\right)\in R^{N\times N}$, $\Gamma_2\in R^{n\times n}$ are defined as $D=\left(d_{ij}\right)\in R^{N\times N}$, $\Gamma_1\in R^{n\times n}$ in network (1) respectively; u_i is the synchronous controller to be designed.

We can see from the model of drive-response neural networks that the topology and node dynamics of the two networks can be different.

Definition 1 [25] The node error of generalized synchronization between systems (1) and (2) is defined as

$$e_i\left(t\right)=y_i\left(t\right)-\varphi_i\left(x_i\left(t\right)\right),\quad i=1,2,\cdots,N\tag{3}$$

where $\varphi_i:R^n\to R^n\ \left(i=1,2,\cdots,N\right)$ is a vector mapping.

If there is a controller u_i, such that $\lim_{t\to\infty}e_i\left(t\right)=0\left(i=1,2,\cdots,N\right)$, then the networks (1) and (2) are said to be generalized synchronized.

To get the generalized synchronization of networks (1) and (2), the controller u_i is designed as follows:

$$\begin{aligned}u_i=&J_{\varphi_i}\dot{x}_i\left(t\right)+C\varphi_i\left(x_i\left(t\right)\right)-Ag\left(\varphi_i\left(x_i\left(t\right)\right)\right)-Bg\left(\varphi_i\left(x_i\left(t-\tau\left(t\right)\right)\right)\right)\\&-c_0\sum_{j=1}^{N}g_{ij}\Gamma_2\left(\varphi_j\left(x_j\left(t\right)\right)\right)-k_ie_i\left(t\right)\end{aligned}\tag{4}$$

$$\dot{k}_i=r_i\left\|e_i\left(t\right)\right\|^2\tag{5}$$

where r_i is an arbitrary positive constant; J_{φ_i} is the Jacobian matrix of the mapping φ_i, $J_{\varphi_i}=\begin{vmatrix}\frac{\partial\varphi_{i1}}{\partial x_{i1}}&\frac{\partial\varphi_{i1}}{\partial x_{i2}}&\cdots&\frac{\partial\varphi_{i1}}{\partial x_{in}}\\\frac{\partial\varphi_{i2}}{\partial x_{i1}}&\frac{\partial\varphi_{i2}}{\partial x_{i2}}&\cdots&\frac{\partial\varphi_{i2}}{\partial x_{in}}\\\vdots&\vdots&\ddots&\vdots\\\frac{\partial\varphi_{in}}{\partial x_{i1}}&\frac{\partial\varphi_{in}}{\partial x_{i2}}&\cdots&\frac{\partial\varphi_{in}}{\partial x_{in}}\end{vmatrix}$, $i=1,2,\cdots,N$.

Under the action of the controller, the error dynamic equation of the system is described as

$$
\begin{aligned}
\dot{e}_i(t) &= \dot{y}_i(t) - J_{\varphi_i}\dot{x}_i(t) \\
&= -C\left[y_i(t) - \varphi_i(x_i(t))\right] + A\left[g(y_i(t)) - g(\varphi_i(x_i(t)))\right] \\
&\quad + B\left[g(y_i(t-\tau(t))) - g(\varphi_i(x_i(t-\tau(t))))\right] \\
&\quad + c_0\sum_{j=1}^{N} g_{ij}\Gamma_2\left[y_j(t) - \varphi_j(x_j(t))\right] - k_i e_i(t)
\end{aligned} \tag{6}
$$

In order to study the problem, the following assumptions and lemma are needed.

Assumption 1 For function $g(x)$, there exists a positive that L such that

$$\|g(y) - g(x)\| \le \sqrt{L}\|y - x\|,$$

for any two vectors $x, y \in R^n$.

Assumption 2 The time varying delay $\tau(t)$ satisfies: $0 \le \dot{\tau}(t) \le \varepsilon < 1$, where ε is a known constant.

These two assumptions are very common in the synchronization of complex networks.

Lemma 1 [28] For any two vectors $x, y \in R^n$, the following formula holds $x^{\mathrm{T}}x + y^{\mathrm{T}}y \ge 2x^{\mathrm{T}}y$.

3. Main Result

In this section, we investigate the generalized synchronization problem of networks (1) and (2). The main result is given by the following theorem.

Theorem 1 If the topological structures of the drive-response neural networks (1) and (2) are known, then they can achieve generalized synchronization under the adaptive controller (4) and updated rule (5).

Proof: Construct the Lyapunov function as follows:

$$V(t) = V_1(t) + V_2(t) \tag{7}$$

where

$$
\begin{aligned}
V_1(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}\frac{1}{r_i}\left(k_i - \bar{k}\right)^2 \\
V_2(t) &= \sum_{i=1}^{N}\frac{\alpha}{1-\varepsilon}\int_{t-\tau(t)}^{t} e_i^{\mathrm{T}}(\theta)e_i(\theta)\mathrm{d}\theta
\end{aligned} \tag{8}
$$

$\bar{k}$ and α are positive constants to be determined.

Calculating the time derivative of $V(t)$ along the error system (6),

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \tag{9}$$

According to (5) and (6), we can get

$$
\begin{aligned}
\dot{V}_1(t) &= 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) + 2\sum_{i=1}^{N}\frac{1}{r_i}\left(k_i - \bar{k}\right)\dot{k}_i \\
&= -2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)C\left[y_i(t) - \varphi_i(x_i(t))\right] - 2\sum_{i=1}^{N}\bar{k}\left\|e_i(t)\right\|^2 \\
&\quad + 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)A\left[g(y_i(t)) - g(\varphi_i(x_i(t)))\right]
\end{aligned}
$$

$$
\begin{aligned}
&+2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) B\left[g\left(y_i(t-\tau(t))\right)-g\left(\varphi_i\left(x_i(t-\tau(t))\right)\right)\right] \\
&+2c_0\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) g_{ij}\Gamma_2\left[y_j(t)-\varphi_j\left(x_j(t)\right)\right] \\
&=W_1+W_2+W_3+W_4
\end{aligned} \tag{10}
$$

In the above equation,

$$
\begin{aligned}
W_1 &= -2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) C\left[y_i(t)-\varphi_i\left(x_i(t)\right)\right]-2\sum_{i=1}^{N}\bar{k}\left\|e_i(t)\right\|^2, \\
W_2 &= 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) A\left[g\left(y_i(t)\right)-g\left(\varphi_i\left(x_i(t)\right)\right)\right], \\
W_3 &= 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) B\left[g\left(y_i(t-\tau(t))\right)-g\left(\varphi_i\left(x_i(t-\tau(t))\right)\right)\right], \\
W_4 &= 2c_0\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) g_{ij}\Gamma_2\left[y_j(t)-\varphi_j\left(x_j(t)\right)\right].
\end{aligned}
$$

Let $\lambda_0=\min\left(c_1,c_2,\cdots,c_n\right)$, $e(t)=\left[e_1^{\mathrm{T}}(t),e_2^{\mathrm{T}}(t),\cdots,e_N^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, then

$$
\begin{aligned}
W_1 &= -2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) C\left[y_i(t)-\varphi_i\left(x_i(t)\right)\right]-2\sum_{i=1}^{N}\bar{k}\left\|e_i(t)\right\|^2 \\
&= -2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) C e_i(t)-2\bar{k}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\
&\le -2\lambda_0\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t)-2\bar{k}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\
&= -2\left(\lambda_0+\bar{k}\right) e^{\mathrm{T}}(t) e(t)
\end{aligned} \tag{11}
$$

By using Lemma 1 and Assumption 1, it's easy to get

$$
\begin{aligned}
W_2 &= 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) A\left[g\left(y_i(t)\right)-g\left(\varphi_i\left(x_i(t)\right)\right)\right] \\
&\le \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) AA^{\mathrm{T}} e_i(t) \\
&\quad +\sum_{i=1}^{N}\left[g\left(y_i(t)\right)-g\left(\varphi_i\left(x_i(t)\right)\right)\right]^{\mathrm{T}}\left[g\left(y_i(t)\right)-g\left(\varphi_i\left(x_i(t)\right)\right)\right] \\
&\le \lambda_{\max}\left(AA^{\mathrm{T}}\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t)+L\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\
&= \left(\lambda_1+L\right) e^{\mathrm{T}}(t) e(t)
\end{aligned} \tag{12}
$$

where $\lambda_1=\lambda_{\max}\left(AA^{\mathrm{T}}\right)$ is the maximum eigenvalue of matrix AA^{T}.

Same as W_2, we have

$$
\begin{aligned}
W_3 &= 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) B\left[g\left(y_i(t-\tau(t))\right)-g\left(\varphi_i\left(x_i(t-\tau(t))\right)\right)\right] \\
&\le \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) BB^{\mathrm{T}} e_i(t)+\sum_{i=1}^{N}\left[g\left(y_i(t-\tau(t))\right)-g\left(\varphi_i\left(x_i(t-\tau(t))\right)\right)\right]^{\mathrm{T}} \\
&\quad \times\left[g\left(y_i(t-\tau(t))\right)-g\left(\varphi_i\left(x_i(t-\tau(t))\right)\right)\right] \\
&\le \lambda_{\max}\left(BB^{\mathrm{T}}\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t)+L\sum_{i=1}^{N} e_i^{\mathrm{T}}\left(t-\tau(t)\right) e_i\left(t-\tau(t)\right) \\
&= \lambda_2 e^{\mathrm{T}}(t) e(t)+L e^{\mathrm{T}}\left(t-\tau(t)\right) e\left(t-\tau(t)\right)
\end{aligned} \tag{13}
$$

where $\lambda_2 = \lambda_{\max}\left(BB^{\mathrm{T}}\right)$.

For convenience of presentation, let $M = G \otimes \Gamma_2$, $\lambda_3 = \lambda_{\max}\left(MM^{\mathrm{T}}\right)$, we obtain

$$
\begin{aligned}
W_4 &= 2c_0 \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) g_{ij}\Gamma_2\left[y_j(t) - \varphi_j\left(x_j(t)\right)\right] \\
&= 2c_0 \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t) g_{ij}\Gamma_2 e_j(t) \\
&= 2c_0\left[e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \cdots, e_N^{\mathrm{T}}(t)\right]\begin{vmatrix} g_{11}\Gamma_2 & g_{12}\Gamma_2 & \cdots & g_{1N}\Gamma_2 \\ g_{21}\Gamma_2 & g_{22}\Gamma_2 & \cdots & g_{2N}\Gamma_2 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N1}\Gamma_2 & g_{N2}\Gamma_2 & \cdots & g_{NN}\Gamma_2 \end{vmatrix} \\
&\quad \times\left[e_1(t), e_2(t), \cdots, e_N(t)\right]^{\mathrm{T}} \\
&= 2c_0 e^{\mathrm{T}}(t)\left(G \otimes \Gamma_2\right) e(t) \\
&= 2c_0 e^{\mathrm{T}}(t) M e(t) \\
&\le c_0\left[e^{\mathrm{T}}(t) MM^{\mathrm{T}} e(t) + e^{\mathrm{T}}(t) e(t)\right] \\
&\le c_0\left[\lambda_{\max}\left(MM^{\mathrm{T}}\right) e^{\mathrm{T}}(t) e(t) + e^{\mathrm{T}}(t) e(t)\right] \\
&= c_0\left(\lambda_3 + 1\right) e^{\mathrm{T}}(t) e(t)
\end{aligned} \tag{14}
$$

The time derivative of $V_2(t)$ can be expressed as

$$
\dot{V}_2(t) = \frac{\alpha}{1-\varepsilon} e^{\mathrm{T}}(t) e(t) - \frac{\alpha\left(1-\dot{\tau}(t)\right)}{1-\varepsilon} e^{\mathrm{T}}\left(t-\tau(t)\right) e\left(t-\tau(t)\right) \tag{15}
$$

Combining (7) - (15) yields

$$
\begin{aligned}
\dot{V}(t) \le{}& -2\lambda_0 e^{\mathrm{T}}(t) e(t) + \left(L - \frac{\alpha\left(1-\dot{\tau}(t)\right)}{1-\varepsilon}\right) e^{\mathrm{T}}\left(t-\tau(t)\right) e\left(t-\tau(t)\right) \\
& + \left(\lambda_1 + L + \lambda_2 + c_0\lambda_3 + c_0 - 2\bar{k} + \frac{\alpha}{1-\varepsilon}\right) e^{\mathrm{T}}(t) e(t)
\end{aligned} \tag{16}
$$

Take $\alpha = L, \bar{k} = \frac{1}{2}\left(\lambda_1 + L + \lambda_2 + c_0\lambda_3 + c_0 + \frac{L}{1-\varepsilon} + 1\right)$, then under the premise of Assumption 2 that (16) can be estimated as

$$
\dot{V}(t) \le -e^{\mathrm{T}}(t) e(t) \tag{17}
$$

It can be found from (17) that $0 \le V(t) \le V(0)$, this together with (7) and (8) signifies $V(t)$ is bounded. We can also obtain $\lim_{t\to\infty}\int_{t-\tau(t)}^{t} e^{\mathrm{T}}(\theta) e(\theta)\,\mathrm{d}\theta \le V(0) - \lim_{t\to\infty} V(t)$. Based on Lyapunov stability theory and LaSalle's invariance principle [29], we have $\lim_{t\to\infty} e_i(t) = 0\ (i = 1, 2, \cdots, N)$. Then it follows from Definition 1 that the drive-response networks (1) and (2) achieve generalized synchronization. The proof is completed.

4. Conclusion

In this paper, the problem of generalized synchronization for drive-response neural networks with time-varying delay and different node dynamics is concerned.

We design an appropriate nonlinear adaptive controller and construct a suitable Lyapunov function so that the desired synchronization is achieved. Because generalized synchronization is a function mapping relationship, the controller of generalized synchronization is complex. How to simplify the controller and make it have a simpler form will be a problem to be studied in the future.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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