



Fast, Ultraluminous X-Ray Bursts from Tidal Stripping of White Dwarfs by Intermediate-mass Black Holes

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Abstract

Two X-ray sources were recently discovered by Irwin et al. that are compact companions to elliptical galaxies showing ultraluminous flares with fast rise (\sim minute) and decay (\sim hour), and with a peak luminosity $\sim 10^{40-41}$ erg s $^{-1}$. Together with two other sources found earlier, they constitute a new type of fast transients that cannot be attributed to neutron stars, but might be due to intermediate-mass black holes (IMBHs; $10^{2-4} M_{\odot}$). The flaring behavior is recurrent for at least two sources. If the flare represents a short period of accretion onto an IMBH during the periastron passage of a donor star on an eccentric (i.e., repeating) or parabolic (non-repeating) orbit, we argue that the flare's rise time corresponds to the duration during which the donor's tidally stripped mass joins a residual disk at the pericenter. This duration is in turn equal to three other timescales: the duration of stripping, the sound crossing time of the donor, and the circular orbit time at the pericenter radius. Only a white dwarf (WD) can have a sound crossing time as short as one minute. Therefore, the donor must be a WD and it was stripped of $\sim 10^{-10} M_{\odot}$ upon each passage at several to tens of Schwarzschild radii from the IMBH. The flux decay corresponds to the viscous drainage of the supplied mass toward the hole. Aided by long-term X-ray monitoring, this type of fast transient would be an ideal target for next-generation gravitational wave detectors.

Key words: accretion, accretion disks – hydrodynamics – stars: black holes – white dwarfs – X-rays: bursts

1. Introduction

There is much evidence for the existence of stellar-mass ($\sim 10 M_{\odot}$) and supermassive ($\sim 10^6$ – $10^9 M_{\odot}$) black holes (BHs), but still no firm evidence for existence of intermediate-mass black holes (IMBHs; $\sim 10^{3-4} M_{\odot}$; Portegies Zwart & McMillan 2002; Baumgardt et al. 2003; Tremou et al. 2018), which fill a gap of the mass range in between. Nevertheless, the search for and identification of them has great impact on our understanding of the seeds and growth history of super-massive BHs (Volonteri et al. 2003; Greene 2012).

Recently, Irwin et al. (2016) found two luminous fast flaring sources in nearby galaxies from a search in archival *Chandra* data. One source is located in a globular cluster (GC) in the galaxy NGC 4636. It brightens within 22 s by a factor of 100 to reach a peak luminosity of 9×10^{40} erg s $^{-1}$, then decays in 1400 s. The persistent emission before and after the flare is at 8×10^{38} erg s $^{-1}$.

The second source is in the elliptical galaxy NGC 5128. It flared five times during a total observation time of 790 ks, yielding an approximate recurrent time of 1.8 days. The flares rise rapidly within 30 s by a factor of 200 to a peak luminosity of 8×10^{39} erg s $^{-1}$, stay in a roughly steady ultraluminous state for ~ 200 s, then decay over 4000 s to the pre-flare level of $\sim 4 \times 10^{37}$ erg s $^{-1}$. The optical counterpart is either a massive GC, or a ultra-compact dwarf companion galaxy of NGC 5128 (Irwin et al. 2016).

The pre-flare and post-flare emission are found to be persistent during all the non-flare observation periods for the two sources. Absorbed power-law fits to the spectra of the persistent emission give photon indices $\Gamma \sim 1.6 \pm 0.5$. The in-flare spectra are more poorly constrained and can be fit either by an absorbed power law (photon indices $\Gamma \sim 1.3$ – 1.6) or by a disk blackbody ($kT \sim 1.3$ – 2.2 keV). No significant spectral

evolution is found, either between the persistent and the flare periods, or during the flares.

Sivakoff et al. (2005) reported two fast flares from an off-center source CXOU J124839.0–054750 in the elliptical galaxy NGC 4697, from two of five *Chandra* observations (~ 40 ks each) of the galaxy. The flares have a peak luminosity of $\sim 6 \times 10^{39}$ erg s $^{-1}$, a duration ~ 70 s, and a count rate ratio of the flare to the persistent emission ~ 90 . The photon counts are few (2–3 for each flare) so the statistical significance is not as strong as those in Irwin et al. (2016). The small number of photons likely causes the flare duration to be underestimated. In addition, Jonker et al. (2013) reported a bright fast flaring source in *Chandra* images of the old elliptical galaxy M86. Following Irwin et al. (2016), we refer to the four sources collectively as fast, ultraluminous X-ray bursts (UXBs), and summarize them in Table 1.

UXBs are unlikely of the same origins as those known bursting phenomena thought to happen to very young and highly magnetized neutron stars, such as soft gamma-ray repeaters, anomalous X-ray pulsars, or types I and II X-ray bursts, for the following reasons given by Irwin et al. 2016. (1) UXBs mostly are found in old stellar population, namely GCs or the compact dwarf companions of elliptical galaxies. (2) Compared with UXBs, the sporadic bursts from those previously known sources either last for too short of a time, or have flare-to-pre-flare flux ratios that are too low. (3) The peak luminosities of UXBs would be super-Eddington for a neutron star. Note that although a handful of ultraluminous X-ray pulsars have been found that break this limit, they are usually thought to possess a high magnetic field strength ($\gtrsim 10^{12-13}$ Gauss) and probably a stable, copious mass supply (e.g., Israel et al. 2017). These two conditions contradict either the oldness of the environment or the transient nature of UXBs.

If the peak luminosity of the flare is limited by the Eddington luminosity L_{Edd} of the BH (though one exception is a beamed

Table 1
Observed Fast Ultraluminous X-Ray Bursts as Possible White Dwarf Tidal Stripping Candidate Events

Source	Host Galaxy	Number of Flares	Rise Time (s)	Decay Time (s)	L_X at Peak (erg s $^{-1}$)	Factor of L_X Increase	Recurrence Time (day) ^a	References ^b
1	Ellip. NGC 4636	1	22	1400	9×10^{40}	~ 100	>4	1.
2	Ellip. NGC 5128	5	30	4000	8×10^{39}	~ 200	~ 1.8	1.
3	Ellip. NGC 4697	2	$\lesssim 70$	$\gtrsim 70$	6×10^{39}	~ 90	~ 1	2.
4	Ellip. M86 (likely)	1	~ 20	$\sim 10^4$	6×10^{42}	~ 600	>3.5	3.

Notes.

^a For single-flare sources, the lower limit is the total observation time; for recurrent flaring sources, it is the total observation time divided by the number of flares, thus a crude estimate.

^b 1. Irwin et al. (2016); 2. Sivakoff et al. (2005); 3. Jonker et al. (2013).

emission), then the BH mass of $M \sim 10^{2-4} M_\odot$ is implied for most of the sources. Each flare emits a total energy of 10^{42-43} erg for the two sources in Irwin et al. (2016), which translates to a total accreted mass of $10^{-11 \sim -10} \eta_{0.1}^{-1} M_\odot$, where $\eta = \eta_{0.1} \times 10\%$ is the radiative efficiency. Note that this mass estimate is a lower limit because the radiative luminosity is probably Eddington-limited, therefore the efficiency could be much lower.

Under the condition of a pure blackbody, the loosely inferred temperatures could also hint at the size of the emission region: $R_{\text{BB}} = [L/(4\pi\sigma T^4)]^{1/2}$. For source 1, $R_{\text{BB}} \approx 5 \times 10^7$ cm, and for source 2, $R_{\text{BB}} \approx 5 \times 10^6$ cm, implying very compact emission regions.

If the flare represents a short period of accretion onto the IMBH during the periastron passage of a donor star on an eccentric or parabolic orbit, we argue that the flare's rise time corresponds to the duration during which the donor's tidally stripped mass returns to the pericenter. This duration is in turn equal to the other three timescales: the duration of the stripping, the sound crossing time of the donor, and the circularly orbital time of the transient disk formed at the pericenter radius. Only a white dwarf (WD) can have a sound crossing time as short as one minute.

We cannot rule out the possibility that the binary comprises a stellar-mass BH and a WD, as the timescales that we will analyze in the next section are all independent of the BH mass. However, for a stellar-mass BH, the peak luminosity would be super-Eddington; to relax this limit one usually needs to invoke beaming of the radiation.

In Section 2 we describe the tidal stripping and relevant physical timescales. We investigate the causes of the flare's flux decay and rise in Sections 3 and 4, respectively, paying particular attention to the interaction of the stripped stream with a residual disk, and the subsequent accretion. We summarize and give further discussion in Section 5.

2. Periastron Tidal Stripping of the Secondary

Consider an IMBH with a companion star orbiting around it on an elliptical orbit, as is illustrated in Figure 1. Let M and M_* be the masses of the BH and the secondary star, respectively. The orbital period P and the semimajor axis a are related as $GMP^2 = 4\pi^2 a^3$, thus, $a = 3 \times 10^{12} M_3^{1/3} P_d^{2/3}$ cm = $10^4 (P_d/M_3)^{2/3} R_S$, where $M_3 = M/10^3 M_\odot$, $P_d = P/1$ day, and R_S is the BH's Schwarzschild radius.

The secondary provides mass to the BH each time it moves to the pericenter whose distance from the BH is R_p , at which the secondary just fills its Roche lobe; i.e., the star's radius R_* is about its Roche lobe size. Therefore (Paczynski 1971;

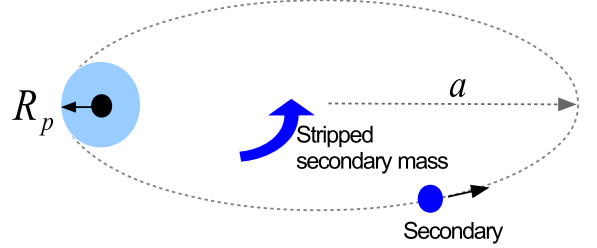


Figure 1. Illustration showing how a secondary on an eccentric orbit about a black hole is tidally stripped at R_p . The stripped material falls back along a smaller orbit, hits the outer edge of a residual disk, and replenishes it with mass. The residual disk is left from previous stripping and replenishment.

Eggleton 1983; Sepinsky et al. 2007)

$$R_p \simeq 2R_* \left(\frac{M}{M_*} \right)^{1/3} \simeq 24 R_* \left(M_3 \frac{0.6M_\odot}{M_*} \right)^{1/3}. \quad (1)$$

In terms of the Schwarzschild radius,

$$R_p = 55 M_3^{-2/3} \frac{R_*}{0.01R_\odot} \left(\frac{0.6M_\odot}{M_*} \right)^{1/3} R_S. \quad (2)$$

Though here and after we normalize the secondary by typical numbers for a WD, the equations are valid for all types of stars.

The duration of the Roche lobe overflow (stripping) is $t_{\text{of}} \simeq R_p/v_p$, where $v_p \simeq (2GM/R_p)^{1/2}$ is the secondary's orbital speed at R_p . From Equation (1), it is easy to see that the duration of the stripping is roughly the internal dynamical timescale (also the sound crossing time) of the secondary star $t_{\text{dyn}} \simeq (G\rho_*)^{-1/2} \simeq [R_*^3/(GM_*)]^{1/2}$, i.e.,

$$t_{\text{of}} \simeq 2 t_{\text{dyn}} \simeq 6 \left(\frac{R_*}{0.01R_\odot} \right)^{3/2} \left(\frac{0.6M_\odot}{M_*} \right)^{1/2} \text{ s}. \quad (3)$$

After the periastron passage, because the stripped matter has a binding energy (with respect to the BH) of $E_{\text{min}} = -GMR_*/R_p^2$, it follows an elliptical trajectory with a semimajor axis of $a_{\text{min}} = 2R_*(M/M_*)^{2/3}$, and a fallback time of

$$t_{\text{fb}} = 2\pi \sqrt{\frac{a_{\text{min}}^3}{GM}} \simeq 18 \left(\frac{M}{M_*} \right)^{1/2} t_{\text{dyn}}. \quad (4)$$

The above is valid under the assumption that the binding energy of the center of mass of the star is close to zero. This condition is satisfied as long as $a_{\text{min}} \ll a$. Within the stripped mass, the spread of binding energy is small, therefore, the spread of t_{fb} is also small: $\delta t_{\text{fb}} < t_{\text{of}}$ (see Appendix).

The fourth timescale is the local circularly orbital timescale at R_p :

$$t_{\text{cir}}(R_p) = 2\pi \sqrt{\frac{R_p^3}{GM}} \simeq 18 t_{\text{dyn}}, \quad (5)$$

Thus, three of the above timescales are of the same order, $t_{\text{dyn}} \sim t_{\text{of}} \sim t_{\text{cir}}(R_p)$. This makes $t_{\text{cir}}(R_p)$ a unique quantity that depends only on the property of the donor star and not on the BH mass at all. For a Sun-like star, $t_{\text{cir}}(R_p) = 0.33$ day. For an evolved star such as the red giant Arcturus with $M_* = 1.1 M_\odot$ and $R_* = 25 R_\odot$, $t_{\text{cir}}(R_p) = 40$ day.

3. Cause of Flux Decay of Bursts

The duration of the mass supply at R_p is approximately equal to the duration of the stripping, and again is approximately equal to the internal dynamical timescale of the donor, t_{dyn} (see Equations (1)–(3)). The closeness of numbers for t_{of} and $t_{\text{cir}}(R_p)$ for a WD to the observed fast rise time (\lesssim minute) of the flares already hints at a WD being the donor. But here let us consider the decay time of the flare first, regardless of this proposition.

The disk’s viscous timescale (representing the time that each mass element spends on its way to BH) at any radius R is

$$t_{\text{vis}}(R) = \frac{t_{\text{cir}}(R)}{2\pi\alpha} \left(\frac{H}{R}\right)^{-2}, \quad (6)$$

where α is the Shakura & Sunyaev viscosity parameter, and H/R is the disk height-to-radius ratio. From Equations (5)–(6) one gets $t_{\text{vis}}(R_p) \simeq 28 \alpha_{0.1}^{-1} (H/R)^{-2} t_{\text{dyn}}$, where $\alpha_{0.1} = \alpha/0.1$. This is the timescale over which the subsequent accretion rate (also the radiative luminosity) from a suddenly supplied mass at R_p decays self-similarly (e.g., Lynden-Bell & Pringle 1974).

If the disk is in the radiatively efficient, geometrically thin regime (Shakura & Sunyaev disk), then $H/R \approx 0.02 (\alpha m)^{-1/10} \dot{m}^{1/5} r^{1/20}$. Here m , \dot{m} and r are BH mass, accretion rate, and radius, normalized by M_\odot , $L_{\text{Edd}}/(0.1c^2)$ and R_S , respectively. In the advective cooling dominated, geometrically thick regime (slim disk), $H/R \approx 1$. The border line between the two regimes is $\dot{m} \sim r/10$ (e.g., Kato et al. 2008). Because the accretion rate near the flare peak is around the Eddington rate, the real H/R is likely between the two limiting values: $0.02 < H/R < 1$.

Therefore, we see that $t_{\text{vis}}(R_p) \gg t_{\text{cir}}(R_p) \sim t_{\text{dyn}}$ because $H/R < 1$. This suggests that the flare decay time is more likely determined by $t_{\text{vis}}(R_p)$, rather than by t_{dyn} . The observed decay time of 10^3 – 10^4 s means a rather short internal dynamical timescale of the donor $t_{\text{dyn}} \sim 100 \alpha_{0.1} (H/R)^2$ s. Therefore, a main-sequence donor is unlikely, as, for instance, the Sun has $t_{\text{dyn}} \approx 1.6 \times 10^3$ s; but the timescale is consistent with a WD being the donor.

4. Causes of Flux Rise of Bursts

Now we return to the rise time. Here we consider two independent scenarios.

4.1. Onset of Accretion near the Innermost Stable Circular Orbit (ISCO)

The disk surface temperature typically drops with radius as $T(R) \propto R^{-p}$, where $p > 0$ (e.g., Kato et al. 2008). Suppose R_X

is a radius in the disk within which the disk is hot enough to be X-ray bright. Therefore, the rise time corresponds to the timescale over which the “head” of the supplied mass accretes within the disk from R_X to the BH, i.e., the viscous timescale at R_X :

$$t_{\text{vis}}(R_X) = 0.14 \frac{M_3}{\alpha_{0.1}} \left(\frac{R_X}{R_S}\right)^{3/2} \left(\frac{H}{R}\right)^{-2} \text{ s}. \quad (7)$$

So if $R_X \approx 6.6 R_S$ and $H/R \approx 0.2$, then $t_{\text{vis}}(R_X) \sim 60$ s, which is consistent with the observed rise time. That is to say, most of the emission during the flare is radiated from close to the ISCO of the BH.

4.2. Stream–Disk Interaction

After returning to the pericenter, the stripped material needs to dissipate its kinetic energy in order to circularize and form a disk. The specific energy to be dissipated is $\sim GM/R_p$. The dissipation is efficient when there is a residual accretion disk left from the previous episode of tidal stripping and mass replenishment. The existence of such a residual disk is supported by the fact that UXB sources show “persistent” emission before and after the flares, and during all the “non-flare” observation periods (see Irwin et al. 2016).

The outer radius of the freshly formed disk is $\sim 2R_p$ if the bound material carries the same specific angular momentum as that of the star. Therefore, the returning stripped stream will collide with the outer disk at R_p , with a relative speed $\simeq 0.4(GM/R_p)^{1/2}$ between the two. The portion of the disk mass that collides can be comparable to the stream’s mass, while the total disk mass might easily exceed the latter. This is because it is a cumulative residual from many previous rounds of mass replenishing and accretion. The collision efficiently dissipates a large portion of the stream’s orbital energy, with an equivalent efficiency (converting the rest-mass energy of the stream to heat) of $\eta \sim 0.001$ (e.g., Equation (2)).

The dissipation heats the interaction site, whose size R_s would be slightly larger than the width of the returning stream, but be smaller than the WD itself (because only its surface layer was stripped). A reasonable estimate would be $R_s \sim 10^7$ cm. This agrees with what was inferred from the spectral data of the two sources (see Section 1).

The returning WD collides with the disk at R_p as well. Because of the star’s strong gravity and larger cross section (compared with the returning stream), this interaction probably scoops away a large chunk of the outer disk material through an extended bow shock in front of the moving star. These shock-heated material might produce a bright optical flare via Bremsstrahlung radiation.

Once the stripped stream joins and replenishes the disk, the disk material drains into the BH on the viscous timescale $t_{\text{vis}}(R_p)$. The enhanced accretion rate (thus, the disk radiative luminosity) also subsides self-similarly on the same timescale (see Section 3).

5. Summary and Discussion

Four fast UXBs have been discovered so far. We identify the flux decay of each flare with the viscous drainage of a suddenly supplied mass that was tidally stripped from a donor by a central IMBH ($\sim 10^{2-4} M_\odot$); the rapidity (\sim hours) of the decay suggests that the donor can only be a WD. The fast

rise (\sim minute) can be interpreted either as the onset of emission from the innermost region of the disk, or as due to the collision between the stripped stream and the outer disk when the former joins the latter at R_p .

The idea of a central IMBH is inspired by the observed peak luminosity of UXBs ($\sim 10^{40-42}$ erg s $^{-1}$). A stellar-mass BH ($\sim 10 M_\odot$) instead cannot be ruled out, but it has the issue of attaining super-Eddington luminosities.

The interval between two recurrent flares must be the eccentric orbital period P of the donor. Independent from the type of the donor and the BH mass, it is straightforward to show that (see Section 2)

$$1 - e \equiv \frac{R_p}{a} = 2 \left(\frac{\pi t_{\text{of}}}{P} \right)^{2/3}, \quad (8)$$

where e is the orbital eccentricity. If we identify the flare rise time with the duration of the stripping (Section 4.2), then for $t_{\text{of}} \approx 1$ minute and $P \approx 1$ day, we get $e \approx 0.97$. However, if instead the flux rise corresponds to the onset of X-ray emission from the innermost region of the disk (Section 4.1), then t_{of} can be longer and the above constraint on e relaxes. Subsequent X-ray monitoring of those UXB sources would be key to constraining the binary parameters and verifying the tidal stripping scenario.

The source reported by Sivakoff et al. (2005) has no clear optical counterpart, and the nearest GC is $1''8$ ($\gtrsim 1$ pc) away. It is possible that this source was ejected from its parent GC at the typical escape speed from a GC (e.g., ~ 10 km s $^{-1}$) ~ 1 Myr ago. The ejection of an IMBH can be achieved by the gravitational recoil resulting from a merger with a stellar-mass BH (e.g., Favata et al. 2004; Fragione et al. 2018). It acquires a WD on its way out either by tidal capture of a single red giant (e.g., Fabian et al. 1975; Kalogera et al. 2004) or by exchange interaction with a binary containing a WD (e.g., Ivanova et al. 2010). Hierarchical triple interaction via the Kozai mechanism is needed to maintain a high eccentricity of the IMBH–WD binary. This triple system probably forms when the latter encounters another ordinary binary before it escapes the GC (e.g., Ivanova et al. 2010).

The optical counterpart to the Jonker et al. (2013) source in the M86 galaxy is very faint, with an absolute magnitude $M_i > 25.6$, roughly corresponding to a bolometric luminosity of $< 10^4 L_\odot$. The galaxy is falling into the Virgo cluster and its gas is being stripped away into a long stream. Moreover, M86 shows signs of a recent wet minor merger with another galaxy SDSS J122541.29+130251.2, whose stars are being ripped off and follow the gas stream. The projected location of the UXB source lies close to the gas and star stream (Jonker et al. 2013). So it is very likely that the parent GC of this UXB was tidally disrupted or stripped during the wet merger (e.g., Kruijssen et al. 2012), leaving its low-mass core (which hosts the UXB) ejected in the direction of the stream.

Tidal disruptions of WDs by IMBHs have been widely studied (Krolik & Piran 2011; MacLeod et al. 2014, 2016; Law-Smith et al. 2017), which differ from the tidal stripping considered here in several ways. First, for parabolic orbits, the event rate of stripping is slightly higher than the disruptions as $R_p \approx 2 R_t$. Second, for elliptical orbits, the system spends several orders of magnitude longer in time during which tidal stripping repeatedly occurs than the time when a disruption occurs (see below).

Third, and the most importantly, the radiation timescales from a WD disruption event shall be much longer. In such a event, the fallback mass rate is extremely super-Eddington ($\sim 10^7 \times$ at peak) and it remains above the Eddington rate for ~ 1 yr. One would expect enormous mass ejection in forms of quasi-spherical outflow during this long period due to energy dissipation from debris stream collision and (later) central accretion. Any high-energy emission from near the BH would be reprocessed by the opaque outflow and the photospheric emission of the latter dominates the observation, similar to a main-sequence stellar tidal disruption event (TDE) by IMBHs (Chen & Shen 2018). Therefore, the emission of a WD TDE will be at lower photon energies (UV to soft X-rays) and last much longer (months to a year). Although the fallback time of the most bound debris (which falls back earliest) is relatively short (~ 10 minutes), it is still very hard to envisage a fast transient could emerge that brightens to L_{edd} , shines for 1 minute and then decays. Moreover, the currently detected UXBs lack any sign of significant spectral or absorption evolution, which disfavors a scenario in which the flare had appeared in the earliest minute of a WD TDE and was quickly obscured by the launching of an outflow.

Zalamea et al. (2010) studied the impact of gravitational wave (GW) emission on the orbit of a WD–IMBH ($10^5 M_\odot$) binary that experiences tidal stripping. They show that the slow decrease of R_p (thus, the increase of stripped mass δM in each passage) experiences two stages. Stage (i) is controlled by the gentle GW emission. The fractional change of R_p during each orbit is $\gamma \equiv -PR_p/R_p \sim 10^{-5} M_5^{2/3}$. Stage (ii) is controlled by the WD mass loss, hence the increase of R_* . Stage (i) lasts longer (roughly $\gamma^{-3/5} \sim 10^3$ orbits) and mass loss is gentle, but the mass loss accelerates in stage (ii) until it reaches total disruption (~ 200 orbits). To scale down to $M = 10^3 M_\odot$, stage (i) would be even longer, $\sim 10^4$ orbits. Due to its dual-signal nature, such systems are interesting targets for next-generation GW detectors, e.g., DECIGO and the Einstein Telescope, with the aid of long-term X-ray monitoring.

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Appendix

Here we take a closer look at the tidal stripping, in order to characterize the mass, depth, and specific energy distribution of the stripped layer. Let $x \ll R_*$ be the depth of the stripped surface layer, as is illustrated in Figure 2. Consider a ring of radius y within the layer at depth x . Each point on the ring is at distance $r = \sqrt{y^2 + (R_* - x)^2}$ from the center of the star. At any fixed depth x , the mass per unit depth is

$$\frac{dM}{dx} = 2\pi \int_0^{\sqrt{R_*^2 - (R_* - x)^2}} \rho(x, y) y dy. \quad (9)$$

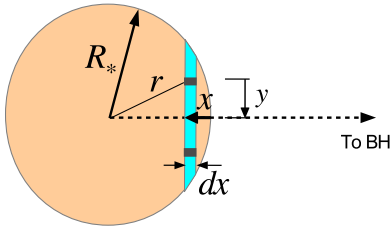


Figure 2. WD donor at the pericenter when its surface layer of a depth x facing the black hole is tidally stripped. Here we neglect tidal deformation.

Changing the variable from y to r by $ydy = rdr$, the integral becomes

$$\frac{dM}{dx} = 2\pi \int_{R_*-x}^{R_*} \rho(r)rdr. \quad (10)$$

Since $x \ll R_*$, we can approximate $r \approx R_*$ and move it out of the integrand. Then we use another depth variable $x' = R_* - r$ to rewrite the above to

$$\frac{dM}{dx} = 2\pi R_* \int_0^x \rho(x')dx'. \quad (11)$$

For simplicity, we assume the surface structure of the WD is described by a polytrope of $P = K\rho^\Gamma$ with $\Gamma = 5/3$, which is the same as in the deeper region where electrons are degenerate and non-relativistic. Once M_* and R_* are given, the value of $K(M_*, R_*)$ is known from numerically solving the Lane–Emden equation. The hydrostatic equilibrium at the surface $dP/dx = GM_*\rho/R_*^2$ gives the density structure there

$$\rho(x) = A\bar{\rho} \left(\frac{x}{R_*} \right)^{3/2}, \quad (12)$$

where $A \simeq 3.8$ and $\bar{\rho} = M_*/(4\pi R_*^3/3)$ is the average density. Therefore,

$$\frac{dM}{dx} = \frac{3}{5}A \frac{M_*}{R_*} \left(\frac{x}{R_*} \right)^{5/2}. \quad (13)$$

The fraction of the total stripped mass is

$$\frac{\delta M}{M_*} = \frac{6}{35}A \left(\frac{x}{R_*} \right)^{7/2}. \quad (14)$$

Equation (14) provides a relation between δM and x , so that one could estimate the depth ratio x/R_* from $\delta M/M$. The observed fluence of the two UXBs in Irwin et al. (2016) suggest about $\sim 10^{-10} M_\odot$ of mass is accreted in each case. This implies that the depth of the stripped layer is $x/R_* \sim 10^{-3}$. An

alternative version of Equation (14) is $\delta M/M_* \propto (x/R_*)^{5/2}$ as was given by Zalamea et al. (2010) who adopted a spherical-shell shape of the stripped layer. There, the stripped layer depth ratio is even smaller, $x/R_* \sim 10^{-5}$, for the same $\delta M/M_*$.

Within the stripped mass, the spread of the binding energy relative to the BH is very small, $\delta E/E \simeq x/R_*$, which means that a very small spread of the returning time $\delta t_{\text{fb}}/t_{\text{fb}} \simeq 3\delta E/(2E) \simeq 3x/(2R_*) \ll 1$. Because $t_{\text{fb}} \sim 9(M/M_*)^{1/2}t_{\text{of}}$ (Equation (4)), that means a small ratio of the spread of returning time over the “length” of the stream, $\delta t_{\text{fb}}/t_{\text{of}} \sim 14(M/M_*)^{1/2}x/R_* \sim 0.6$ for $M/M_* = 10^{3.2}$ and $x/R_* = 10^{-3}$. Therefore, the duration over which the stream of the stripped matter returns to the pericenter and collides with the residual disk is set by the duration of stripping t_{of} .

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