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Gaussian Optics Analysis for Human Eyes with Application for Vision Corrections

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Aims: To derive formulae and analyze the roles of ocular components of human eye on the refractive power in various applications.
Study Design: Gaussian optics analysis.
Place and Duration of Study: Taipei, Taiwan, between May 2015 and August 2016.
Methodology: An effective eye model is introduced by the ocular components of human eye including refractive indexes, surface radius (r1, r2, R1, R2) and thickness (t,T) of the cornea and lens, the anterior chamber depth(S1) and the vitreous length (S2). Gaussian optics is used to calculate the change rate of refractive error per unit amount of ocular components of a human eye.
Results: For typical corneal and lens power of 42 and 21.9 diopters, the rate function defined by the change of refractive error (De) due to the change of ocular components, or Mj =dDe/dQj, with j=1 to 6 for Qj= r1, r2, R1, R2, t, T are calculated for a 1% change of Qj M1=+0.485, M2=-0.063, M3=+0.053, M4=+0.091, M5=+0.012,and M6=-0.021 diopters. For 1.0 mm increase of S1 and S2, the rate functions are: M7=+1.35, and M8=-2.67 diopter/mm.
Conclusion: Using Gaussian optics, we have derived analytic formulas for the change of refractive power due to various ocular parameter changes. These formulas provide the amount of refractive error corrections in various applications including laser in situ keratomileusis (LASIK) surgery and scleral ablation for accommodation.

Keywords: Gaussian optics; human eye; ocular components; refractive errors; vision correction.

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1. INTRODUCTION

Gaussian optics [1,2] has been used for the calculations of intraocular lens (IOL) power, accommodation amplitude in IOL and human natural lens and the refractive state of human eyes [1-6]. A complete optical description of a human eye should include its 12 ocular parameters, 4 refractive indexes, 4 surface radius and 2 thickness (for cornea and lens), the anterior chamber depth and the vitreous length (or axial length). The roles of these parameters on the refractive power of an eye have been reported only partially [2,3]. This study will derive analytic formulas for the change rate of refractive error per unit amount of selected ocular components.

A conversion function defined by the ratio between the refractive error change and the lens power change is used to calculate the rate functions. The roles of the surface radius of the cornea and lens on the refractive error, the competing factors of anterior chamber depth (resulting to hyperopic shift) and posterior chamber depth (resulting to myopic shift) are discussed. Finally, various new applications related to the formulas developed in this paper including laser in situ keratomileusis (LASIK) surgery, corneal cross linking (CXL) procedure, femtosecond laser surgery and laser scleral ablation for accommodation are presented in the manuscript. The analytic formulas developed in this paper provide useful clinical guidance for vision corrections in various applications.

2. MATERIALS AND METHODS

2.1 The Effective Eye Model

By Gaussian optics theory (or paraxial ray approximation along the axial axis), the refractive error (De) as a function of the system effective focal length (EFL) (F), axial length (L) and position of the system second principal plane (L2) as follow [1,2].

$$De = 1000\,[n1/(L-L2) - n1/F], \qquad (1)$$

where n1 is the refractive index of the aqueous humor. F is the system EFL defines the system total power D=1000n1/F (D in diopter, F in mm) which is determined by the corneal (D1) and lens power (D2) as follows [2,3].

$$D = D1 + D2 - S(D1D2)/(1000n1), \qquad (2.a)$$

$$D1 = 1000\,[(n3-1)/r1 - (n3-n1)/r2] + bt, \qquad (2.b)$$

$$D2 = 1000\,[(n4-n1)/R1 + (n4-n2)/R2] - aT, \qquad (2.c)$$

where nj (j=1, 2, 3, 4) are the refractive index for the aqueous, vitreous, cornea and lens, respectively. The anterior and posterior radius of curvatures (in the unit of mm) of the cornea and lens are given by (r1, r2) and (R1, R2), respectively, where the only concave surface R2 is taken as its absolute value in this study. Finally, S is the effective anterior chamber depth, related to the anterior chamber depth (ACD), S1, by S=S1+P11+0.05 (in mm), where P11 is the distance between the lens anterior surface and its first principal plane, and 0.05 mm is a correction amount to include the effect of corneal thickness (assumed to be 0.55 mm) [2,3]. The thickness terms in Eq.(2.b) and (2.c) are given by b=11.3/(r1r2), a=4.97/(R1R2) for refractive indexes of n1 = n2 = 1.336, n3 = 1.377 and n4 = 1.42; and t and T are the thickness of the cornea and lens, respectively.

As shown in Fig. 1, we have derived the equation [3] L-L2=X+ SF/f, with X=L-S-aT+0.05, where aT and 0.05 mm are the correction terms due to lens and cornea thickness. Eq.(1) may be rewritten in an effective eye model equation [3].

$$De = Z^2\,[1336/X - D1/Z - D2] \qquad (3.a)$$

$$Z = 1 - S/f \qquad (3.b)$$

where f (in mm) is the EFL of the cornea given by f=1336/D1, and the nonlinear term k is about 0.003 calculated from the second-order approximation of SF/(1336f). The nonlinear term may also be derived from the IOL power formula [5]. We note that in Eq. (3), X, Z, S and f are in the unit of mm; D1, D2 and De are in the unit of diopter; and the 1336 is from 1000x1.366 in our converted units.

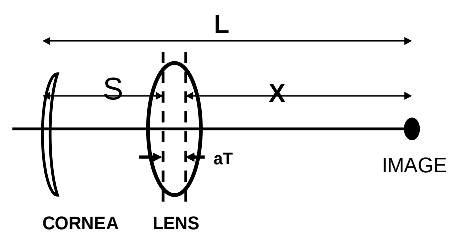

Fig. 1. An effective eye model [3] defined by the power of the cornea and lens. Also shown are the parameters of S and X which is related to the axial length by L=S+X+aT - 0.05 (mm)

2.2 Derivation of the Rate Functions

To find the change of refractive error (De) due to the change of Qj, we further define Qj=(r1, r2, R1, R2, t, T, S1, S2) with j=(1 to 8), respectively. The ACD (S1) and vitreous length (S2) are related to the axial length by L=S1+S2+T. The derivative of the refractive error (De) with respect to these ocular parameter change (Qj) given by Mj=dDe/dQj, defines the rate function, or the change of De per unit amount change of Qj, where the standard notation "d" for "derivative" is used in this study.

In general, under the second-order approximation including the contributions from both n1/(L-L2) and (n1/F) in Eq.(1), one shall rigorously calculate the derivative dDe=Mj(dQj) based on Eq.(1). The complexity of this method is due to the nonlinear dependence of L2 on the ocular parameters [1-3]. In this study, an alternative method is described as follows. For cornea related ocular parameters Qj (with j=1,2,5), the rate function Mj may be calculated by the corneal conversion function C_1 = (dDe/dD1), such that

$$Mj = dDe/dQj = C_1 (dDe/dD1). \quad (4)$$

On the other hand, the lens related parameters Qj (with j=3,4,6) , the rate function may be calculated by a lens conversion function C_2 = (dDe/dD2),

$$Mj = dDe/dQj = C_2 (dDe/dD2). \quad (5)$$

It may be calculated, from Eq. (3.a) that C_1=1.0, for initial De=0. In other words, the corneal power change is 100% converted to the system power or refractive error change. Similarly the lens conversion function given by $C_2= Z^2$, can be derived by taking the derivative of De in Eq. (3.a).

Using Eq. (2), (3), (4) and (5) analytic formulas for the rate function for the surface curvatures and thickness of the cornea and lens are derived by Mj=dDe/dQj, with Qj (j= 1 to 4, for r1, r2, R1 and R2, respectively), and Q5=t, Q6=T as follows.

$$M1 = +378/r1^2, \quad (6.a)$$

$$M2 = -41/r2^2, \quad (6.b)$$

$$M3 = +82.75\, C_2/R1^2, \quad (6.c)$$

$$M4 = +82.75\, C_2 /R2^2, \quad (6.d)$$

$$M5 = 11.3 / (r1r2), \quad (6.e)$$

$$M6 = +4.97\, C_2/(R1R2). \quad (6.f)$$

where we had used the refractive indexes nj=(1.336, 1.336, 1.3371, 1.42) for the aqueous, vitreous, cornea and lens, respectively.

The rate function for S1 and S2, defined by M7=dDe/dS1 and M8=dDe/dS2, were previously derived and given by [3,4].

$$M7= 1336\, (1/F^2 – 1/f^2), \quad (7.a)$$

$$M8= - 1336/F^2, \quad (7.b)$$

where f and F (both in mm) are the corneal and system EFL given by f=1336/D1 and F=1336/D.

Another set of useful parameter is the rate functions due to the refractive index (n1,n2,n3,n4) change which may be easily formulated by taking the derivative of Eq. (3) using a alternative technique. For example, m1=dDe/dn1 may be calculated by two steps. First step is to calculate M9=dD1/dn1 + dD2/dn1, the second step is to multiply C_2 to the second term of M1 to obtain Mj=dDe/dQj, with Q(j=9,10,11,2) for nj (j=1,2,3,4), respectively. We obtain

$$M9= (dD1/dn1)+ C_2\, (dD2/dn1).$$

$$= 1000\, (1/r2 – C_2/R1) \quad (8.a)$$

Similarly for n2, n3 ad n4, we obtained

$$M10 = - 1000C_2/R2, \quad (8.b)$$

$$M11= - 1000\, (1/r2 – 1/r1), \quad (8.c)$$

$$M12 = -1000\, C_2\, (1/R1 + 1/R2) \quad (8.d)$$

3. RESULTS AND DISCUSSION

3.1 The Rate Functions

By using a set of typical ocular parameters [2]: refractive indexes nj (i=1 to 4) =(1.336, 1.336, 1.3771, 1.42), (r1, r2)=(7.8, 6.5) mm, (R1, R2)=(10.2, 6.0) mm, thickness (t, T)=(0.55, 4.0) mm and S=6.0, S1=3.5 and S2=16.0 mm, or an axial length of L=3.5 + 16 + 4 = 23.5 mm, the corneal and lens power are calculated D1=42 diopter, D2=21.9 diopter and total power, from Eq.(2.a), D=D1+0.811D2=59.8 diopter, The rate function Mj (j=1 to 6) are calculated for a 1%

change of r1, r2, R1, R2, t, T : (in diopters) M1=+0.485, M2=-0.063, M3=+0.053, M4=+0.091, M5=+0.012,and M6=-0.021 diopters.

For 1.0 mm increase of S1 and S2, the rate functions are: M7=+1.35, and M8=-2.67 diopter/mm. Furthermore, for each 1.0 diopter increase of corneal and lens power, the rate functions are 1.0 and 0.66 diopter, respectively, for a typical value of effective ACD, S=6.0 mm and corneal power of 43 diopters. We shall note that the above values of Mj depend on the choices of the ocular parameters and may vary 10% - 15% from the typical values chosen. Our calculated data are consistent with that of Ref. 2 and raytracing method [6,7].

The increase of radius of curvature of the cornea and lens (r1, r2, R1, R2) all result in hyperopic shift, except the change of the posterior surface of the lens (R2) having a myopia shift, since it is the only concave surface and all other three surfaces (r2, R1, R2) are convex. Furthermore, the effect due to anterior corneal surface change is the dominant one, where M1 is about 8 times of M2 and M3, and 5 times of M4, as shown by Eq. (6). This may be easily realized from Eq. (2.b) that (n3-1) is much higher than the other terms, such as (n3-n1) and (n4-n1). Therefore reshaping of lens surface is much less efficient than that of cornea. We will discuss more later in femtosecond laser procedure.

The increase of S1 results in a hyperopia shift (HS), whereas S2 results in a myopia shift (MS), where M8 is about two times of M7 which has two competing terms as shown by Eq. (7.a). The rather high change rate M12=-2.67 (D/mm) has significant impact on the onset of emmetropization and myopia which are governed by the correlation among the growth of axial length (L=S1+S2+T) and the power decrease of the cornea and lens when an eye grows [3]. The change rate M7 having a lower value than M8 can be analyzed as follows.

The competing between the MS (due to the increase of ACD, S1) and the HS (due to the associate decrease of S2 for a fixed axial length L=S1+S2+T) results in a net hyperopic-shift, because the hyperopic component is always the dominant one, since the corneal power (D1) is always less than the total system power (D) or F<f in Eq. (5.a). This new finding based on the analytic formula of Eq. (5) has not been explored before, in addition to the newly introduced conversion function.

The hyperopic shift due to the increase of S1 is equivalent to a myopic-shift when S1 decreases, or a forward movement of the lens. This feature is important for presbyopia accommodation which is contributed by two components: the lens curvature decrease and the lens forward movement [3,4]. The lens forward movement is also the main feature in an accommodative IOL and our formulas, Eq. (7) for M7 and M8 provide the amount of accommodation.

3.2 Clinical Applications

Two examples of applications related to the formulas developed in this paper, including laser in situ keratomileusis (LASIK) surgery and scleral ablation for accommodation are presented as follows. More clinical applications will be presented else where.

3.2.1 LASIK surgery [8]

LASIK is a procedure where one diopter correction only requires an ablation depth about 8 to 11 microns of the corneal central thickness [6] or a corresponding change of r1 about 0.16 mm based on Eq. (6.a). It is important to know that the corneal power change is 100% converted to the system power or refractive error change, as demonstrated by our cornea conversion factor C_1. We should also note that the refractive error (De) defined on the corneal plan is the same as that of a contact lens. However, a conversion formula is needed when it is translated to a spectacle power Ds, given by De= Ds/ [1 - V Ds], where V is a vertex distance about 12 mm.

The central ablation depth for a 3-zone myopic correction is given by [8].

$$H'(3\text{-zone}) = RH(\text{single-zone}), \quad (9.a)$$

$$H(\text{single-zone})=*DW2/3)(1+C) \quad (9.b)$$

where W is the diameter of the outer ablation zone having a typical value of 6.5 to 7.5 mm; C is a nonlinear correction term given by C= 0.19 $(W/r1)^2$, r1 is the corneal anterior radius of curvature. For examples, for r1=7.8 mm, (or a K-reading of K=43.2 D), C = (11.2, 13.2, 16.5)% for W =(6.0, 6.5, 7.00 mm. The reduction factor R=(0.70 to 0.85) depending on the algorithms used. For example, comparing to a single zone with W=6.5 mm, a 3-zone depth will reduces to 71.6% (or R = 0.716) when an inner zone 5.5 mm and an outer zone 6.5 mm are used.

3.2.2 Presbyopia treatment [9]

Scleral laser ablation and band expansion have been used to increase the space of the ciliary-body and zonus such that accomondation is improved by two components [9]: the lens translation and the lens shaping which are given by, respectively, M7 and M3. For older and/or harder lens, the accommodation is mainly attributed by the lens translation (or S1 change), whereas lens shaping dominates the power change in young or soft lens. It was known that change of the rear surface of the lens is about one-third of the front surface during accommodation [10], our formulas Eq. (6.c) and (6.d) shows that the contribution from R2 is about the same as that of R1, because of R2 (6.0 mm) <R1(10.2 mm), and M4=2.9 M3, for the same change of curvature, dR1= dR2.

4. CONCLUSION

Using Gaussian optics, we have derived analytic formulae for the change of refractive power due to various ocular parameter changes. These formulae provide the amount of refractive error corrections in LASIK and scleral ablation for accommodation. More clinical applications such as corneal cross linking, femtosecond laser surgery and accommodative IOL will be presented else where.

CONSENT

It is not applicable.

ETHICAL APPROVAL

It is not applicable.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Pedrotti LS, Pedrotti F. Optics and vision. Liper Saddle River (NJ), Prentice Hall; 1998.
2. Atchison DA, Smith G. Optics of the human eye. Woburn (MA), Butterworth Heinemann; 2000.
3. Lin JT. Analysis of refractive state ratios and the onset of myopia. Ophthal Physiol Opt. 2006;26:97-105.
4. Lin JT, Jiang MS, Hong YL, et al. Analysis and applications of accommodative lenses for vision corrections. J Biomed Optics. 2010;16:018002-018002-5.
5. Garg A, Lin JT, et al, editors. Mastering IOLs: Principles, techniques and innovations, New Deli (India), Jaypee Brothers; 2007.
6. Nawa Y, Ueda T, Nakasuka, et al. Accommodation obtained per 1.0 mm forward movement of a posterior chamber IOL. J Cataract Refract Surg. 2003;29: 2069-2072.
7. Ho A, Manns F, Pham T, et al. Predicting the performance of accommodating IOL using ray tracing. J Cataract Refract Surg. 2006;32:129-136.
8. Garg A, Lin JT, et al, editors. Mastering the Advanced Surface Ablation Techniques. New Deli (India), Jaypee Brothers; 2008.
9. Lin JT, Mallo O. Treatment of presbyopia by infrared laser radial sclerectomy. J. Refract Surg. 2003;19:465-467.
10. Rosem AM, Denham DB, Fernandez V, et al. *In vitro* dimemnsions and curvatures of human lenses. Vision Res. 2006:46: 1006-1009.

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