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On the Effect of Seasonal Averages and Standard Deviations on Buys-Ballot Estimates of Time Series Components in the Presence of Missing Values

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

In this study, we consider the effect of seasonal averages and seasonal standard deviations on Buys-Ballot estimates of time series components in the presence of missing values. The emphasis is to compare seasonal averages with seasonal standard deviations in the presence and absence of missing values using real life example. The methods adopted are Mean Imputation (MI), Regression Imputations (RI) and Buys-Ballot Procedure (BBP) for estimating missing values in time series data. Result of this analysis shows that, the differences between Seasonal averages and seasonal standard deviations with and without missing values have insignificant effect on the Buys-Ballot estimates of time series components.

Keywords: Model structure; trend cycle component; missing data; seasonal indices; trend parameter; buys-ballot table.

1. INTRODUCTION

The analysis of time series, a problem usually encountered in data collection is a missing values. Missing values may be virtually impossible to get, either because of time or cost constraint. To obtain estimates of these values, there are available options to the researcher. One of the options is to replace them by mean of the data. The missing values may be replaced

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with naïve forecast or with the average of the last two known observations that bound the missing value Almed [1]. Missing observation can lead to wrong conclusions about time series data. The process of substitution of missing observations may introduce inaccuracies. It can lead to inaccurate results, forecast and errors or data skews can proliferate across subsequent runs causing a large cumulative error effect.

Brockwell and Davis [2] discussed the option that missing data at the beginning or end of the series are simply ignored while intermediate missing data are seen as problems in the input time series. Therefore, they observed that, interpolates values using interpolation algorithm linear, polynomial, smoothing, spline and filering.

Cheema [3] gave comparison of different methods of handling missing data. They include mean imputation, regression imputation, maximum likelihood imputation, multiple imputation and listwise deletion.

As the literature reveals, missing observations in time series has actually attracted many research attention. There are several approaches to determine missing observations, a good example is the estimation methods proposed by Iwueze et al. [4]. According to them, three methods include; Row Mean Imputation (RMI), Column Mean Imputation (CMI) and Decomposition Without the Missing Values (DWMV). They added that, Decomposition without the Missing Values (DWMV) yielded best in terms of accuracy measures estimates of the missing values when compared with other existing methods. Therefore, they recommended that, the Decomposition without the Missing Values (DWMV) method be used in estimating missing values in time series analysis when one observation is missing a time in the Buys-Ballot table. Therefore the ultimate objective of this study is to compare seasonal averages with seasonal standard deviations in the presence and absence of missing values using real life example. Specific objectives are 1) to estimate the missing values of the monthly number of registered accidents over the period under investigation. 2) to estimate trend parameters and seasonal indices with missing values. 3) to estimate trend parameters and seasonal indices without missing values. Based on the results, the rationale for this study is actually to add to existing literature, by providing analyst the functional relationship between seasonal averages and seasonal standard deviation and the effect on Buys-Ballot estimates of time series components.

2. METHODOLOGY

The methods used in this study are 1) the Buys-Ballot Procedure (BBP), 2) the Mean (MI) Imputation, 3) the Regression Imputation (RI). The study series is arranged in a Buys-Ballot table with m periods and s seasons. For details of Buys-Ballot table, see Iwueze et al. [4], Akpanta and Iwueze [5], Nwogu et al. [6], Dozie et al. [7], Dozie and Ihekuna [8], Dozie and Ibebuogu [9], Dozie and Ijeomah [10], Dozie and Nwanya [11], Dozie [12].

2.1 Mean Imputation (MI)

Mean imputations is given by

$$MI = \hat{X}_{(i-1)s+j} = \frac{1}{(i-1)s+j-1}\left[X_1 + X_2 + X_3 + \ldots + X_{(i-1)s+j}\right] = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (1)$$

2.2 Regression Imputation (RI)

Regression imputation is given by

$$\hat{X}_{(i-1)s+j} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}\left[(i-1)s+j\right]^2 \quad (2)$$

2.3 Estimation of Trend Parameters and Seasonal Indices

Iwueze and Nwogu [13] provided estimation for trend and seasonal indices shown in equations (3), (4), (5), (6), and (7) below:

$$\hat{a} = a^{|} + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \tag{3}$$

$$\hat{b} = \frac{b^{|}}{s} + \hat{c}(s-1) \tag{4}$$

$$\hat{c} = \frac{c^{|}}{s^2} \tag{5}$$

$$\hat{S}_j = \bar{X}_{.j} - d_j \tag{6}$$

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}\,j^2 \tag{7}$$

2.4 Estimation of Missing Data in the Transformed and Untransformed Time Series

The transformed estimated missing data is given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s + j] + \hat{c}[(i-1)s + j]^2 + \hat{S}_j \tag{8}$$

The estimate of the missing data in the untransformed series.

$$Untransformed \quad \hat{X}_{ij} = e^{\hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 + \hat{S}_j} \tag{9}$$

3. REAL LIFE EXAMPLE

This section contains real life data example to illustrate the methods of mean and regression imputations in estimating trend parameters and seasonal indices in the presence and absence of missing values. One hundred and eight points (108) of reported cases of registered road accidents from the Federal Road Safety Commission (FRSC) in Owerri, Imo State, Nigeria from January, 2013 to December, 2021 are considered in which these points have three (3) missing values. One hundred and five (105) observed values are shown in Appendix A. The time plots of actual and transformed series with missing data are given in Figs. 3.1, 3.2, 3.3 and 3.4. The amplitude of Fig. 3.1 appears to have increased in the later year indicating that the variance is not constant, suggesting that the data requires transformation to stabilize the variance. The natural logarithm of the periodic and standard deviation are given in Table 1. The data is transformed by using the natural logarithm of the one hundred and eight (108) observations by the method of Akpanta and Iwueze [5].

3.1 Estimates with Missing Observations

The estimates with missing observations is given as;

$$\bar{X}_i = 3.437 - 0.0107t + 0.0401t^2 \tag{10}$$

Where, $a^{|} = 3.437,\ b^{|} = -0.0107\ \ c^{|} = 0.0401$

Using (3), (4) and (5) we obtain

$\hat{a} = 3.5563,\ \hat{b} = 0.024,\ \hat{c} = 0.0003,$

$d_j = \bar{X}_{.j} - \hat{S}_{.j} = \bar{X}_{.j} - 5.7163 - 0.0003_{j^2}$

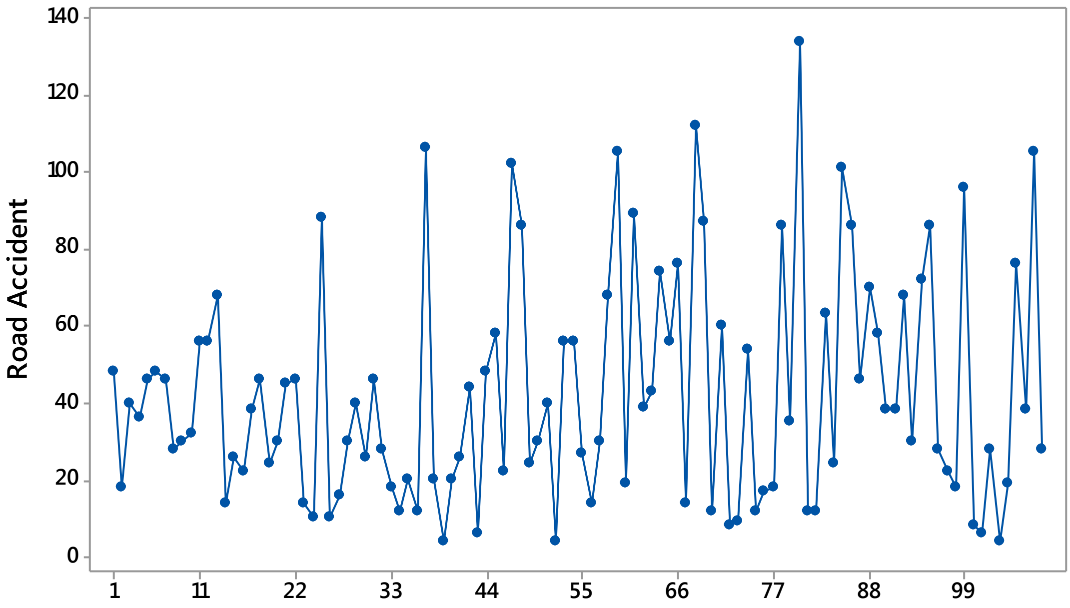

Fig. 3.1. Time plot of actual series with missing observations

Table 1. Natural logarithm of periodic averages and standard deviations with missing values

$\bar{X}_{i.}$	$Log_e \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e \hat{\sigma}_{i.}$
40.33	3.70	11.75	2.46
31.92	3.46	17.13	2.84
28.83	3.36	21.77	3.08
45.20	3.81	35.90	3.58
39.42	3.67	27.89	3.33
55.83	4.02	33.56	3.51
39.70	3.68	38.50	3.65
60.08	4.10	24.18	3.19
37.30	3.62	35.10	3.56

Table 2. Estimates of seasonal indices with missing values

j	$\bar{X}_{.j}$	$Adj\ \hat{S}_j$
1	3.8670	1.4410
2	3.2470	-0.2443
3	3.2700	-0.2983
4	3.1030	-0.0366
5	3.4670	-0.0798
6	3.8360	-0.0555
7	3.0160	-0.3158
8	3.7060	-0.3177
9	3.5830	-0.1922
10	3.3300	-0.5083
11	4.0320	-0.3951
12	3.1260	0.9294

3.2 Estimates without Missing Observations

Here, estimates without missing observations is given as;

$$\bar{X}_i = 3.437 - 0.0108t + 0.0404t^2 \qquad (11)$$

Where, $a^{|} = 3.437,\ b^{|} = -0.0108\ c^{|} = 0.0404$

Using (3), (4) and (5) we obtain

$$\hat{a} = 3.5563,\ \hat{b} = 0.024,\ \hat{c} = 0.0003,$$

$$d_j = \bar{X}_{.j} - \hat{S}_{.j} = \bar{X}_{.j} - 5.7163 - 0.0003_{j^2}$$

Table 3. Estimates of seasonal indices without missing values

j	$\bar{X}_{.j}$	$Adj\ \hat{S}_j$
1	3.8670	1.4410
2	3.2470	-0.2443
3	3.2700	-0.2983
4	3.1030	-0.0366
5	3.4670	-0.0798
6	3.8360	-0.0555
7	3.0160	-0.3158
8	3.7060	-0.3177
9	3.5830	-0.1922
10	3.3300	-0.5083
11	4.0320	-0.3951
12	3.1260	0.9294

Table 4. Transformed series of natural logarithm of periodic averages and periodic standard deviations with missing values

$\bar{X}_{i.}$	$Log_e \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e \hat{\sigma}_{i.}$
3.6510	1.2950	0.3335	-1.0981
3.3190	1.1997	0.5810	-0.5430
3.1600	1.1506	0.6360	-0.4526
3.4150	1.2282	1.0490	0.0478
3.4050	1.2252	0.8530	-0.1589
3.7350	1.3231	0.8770	-0.1312
3.2990	1.1936	0.8920	-0.1143
4.0140	1.3898	0.4340	-0.8347
3.1700	1.1537	1.0520	0.0507

Table 5. Transformed series of natural logarithm of seasonal averages and seasonal standard deviations with missing values

$\bar{X}_{.j}$	$Log_e \bar{X}_{.j}$	$\hat{\sigma}_{.j}$	$Log_e \hat{\sigma}_{.j}$
3.8670	1.3525	0.8620	-0.1485
3.2470	1.7777	0.6880	-0.3740
3.2700	1.1848	0.9350	-0.0672
3.1030	1.1324	0.9460	-0.0555
3.4670	1.2433	0.7390	-0.3025
3.8360	1.3444	0.4020	-0.9113
3.0160	1.1039	0.8940	-0.1120
3.7060	1.3100	0.7770	-0.2523
3.5830	1.2762	0.6520	-0.4277
3.3300	1.2030	0.7290	-0.3160
4.0320	1.3943	0.7360	-0.3065
3.1260	1.1398	0.7810	-0.2472

Table 6. Differences in the period averages with and without missing values

Period i	$\bar{X}_{i.(1)}$	$\bar{X}_{i.(2)}$	$\bar{X}_{i.(1)}$ - $\bar{X}_{i.(2)}$
1	3.6510	3.6510	0
2	3.3190	3.3190	0
3	3.1600	3.1600	0
4	3.4150	3.4150	0
5	3.4050	3.4000	0.0050
6	3.7550	3.7550	0
7	3.2990	3.2901	0.0089
8	4.0140	4.0111	0.0029
9	3.1700	3.1700	0

Table 7. Differences in the seasonal averages with and without missing values

Season j	$\bar{X}_{.j(1)}$	$\bar{X}_{.j(2)}$	$\bar{X}_{.j(1)}$ - $\bar{X}_{.j(2)}$
1	3.8670	3.8670	0
2	3.2470	3.2470	0
3	3.2700	3.2700	0
4	3.1030	3.1011	0.0019
5	3.4670	3.4670	0
6	3.8360	3.8360	0
7	3.0160	3.0160	0
8	3.7060	3.7060	0
9	3.5830	3.5822	0.0008
10	3.3300	3.3300	0
11	4.0320	4.0303	0.0017
12	3.1260	3.1260	0

Table 8. Differences in the periodic standard deviations with and without missing values

Period i	$\hat{\sigma}_{i.(1)}$	$\hat{\sigma}_{i.(2)}$	$\hat{\sigma}_{i.(1)}$ - $\hat{\sigma}_{i.(2)}$
1	0.3335	0.3335	0
2	0.5810	0.5810	0
3	0.6360	0.6360	0
4	1.0490	1.0490	0
5	0.8630	0.8613	0.0017
6	0.8770	0.8770	0
7	0.8920	0.8911	0.0009
8	0.4340	0.4309	0.0031
9	1.0520	1.0520	0

3.3 Estimation of Transformed Missing Observations

Using (8)

$$\text{Transformed value } \hat{X}_{5,4} = 3.7062 + (-0.0320)[(5-1)12+4] + 0.0003[(5-1)12+4]^2 + (-0.0366)$$

$$= 2.8534$$

$$Transformed\ value\ \hat{X}_{7,9} = 3.7062 + (-0.0320)[(7-1)12+9] + 0.0003[(7-1)12+9]^2 + (-0.1922)$$
$$= 2.8903$$

$$Transformed\ value\ \hat{X}_{8,11} = 3.7062 + (-0.0320)[(8-1)12+11] + 0.0003[(8-1)12+11]^2 + (-0.3951)$$
$$= 2.9786$$

3.4 Estimation of Untransformed Missing Observations

Using (9)

$$Untransformed\ value\ \hat{X}_{5,4} = e^{2.8534} = 17.3466 \approx 17$$

$$Untransformed\ value\ \hat{X}_{7,9} = e^{2.8903} = 17.9987 \approx 18$$

$$Untransformed\ value\ \hat{X}_{8,11} = e^{2.9786} = 19.6602 \approx 20$$

Table 9. Differences in the seasonal standard deviations with and without missing values

$Season\ j$	$\hat{\sigma}_{.j(1)}$	$\hat{\sigma}_{.j(2)}$	$\hat{\sigma}_{.j(1)}$ - $\hat{\sigma}_{.j(2)}$
1	0.8620	0.8620	0
2	0.6880	0.6880	0
3	0.9350	0.9350	0
4	0.9460	0.9423	0.0037
5	0.7390	0.7390	0
6	0.4020	0.4020	0
7	0.8940	0.8940	0
8	0.7770	0.7770	0
9	0.6520	0.6512	0.0008
10	0.7290	0.7290	0
11	0.7360	0.7355	0.0005
12	0.7810	0.7810	0

Table 10. The Deviations of seasonal averages from the overall averages $\bar{X}_{.j} - \bar{X}_{..}$

$Seasons\ j$	$\bar{X}_{.j}$	$\bar{X}_{.j} - \bar{X}_{..}$
1	3.8670	0.4017
2	3.2470	-0.2183
3	3.2700	-0.1953
4	3.1030	-0.3623
5	3.4670	0.0017
6	3.8360	0.3707
7	3.0160	-0.4493
8	3.7060	0.2407
9	3.5830	0.1177
10	3.3300	-0.1353
11	4.0320	0.5667
12	3.1260	-0.3393

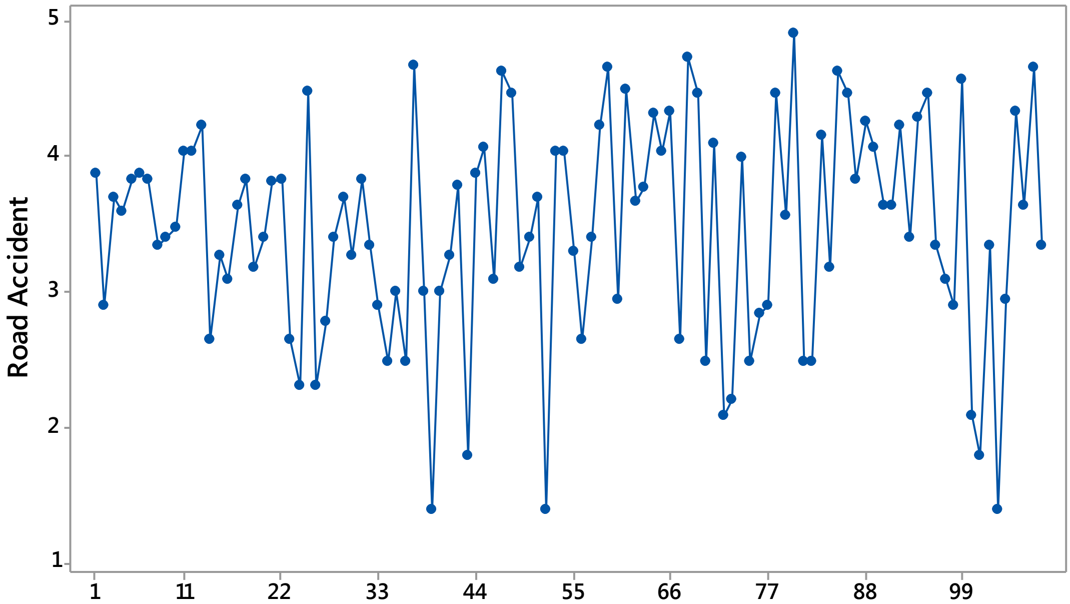

Fig. 3.2. Time plot of transformed series with missing observations

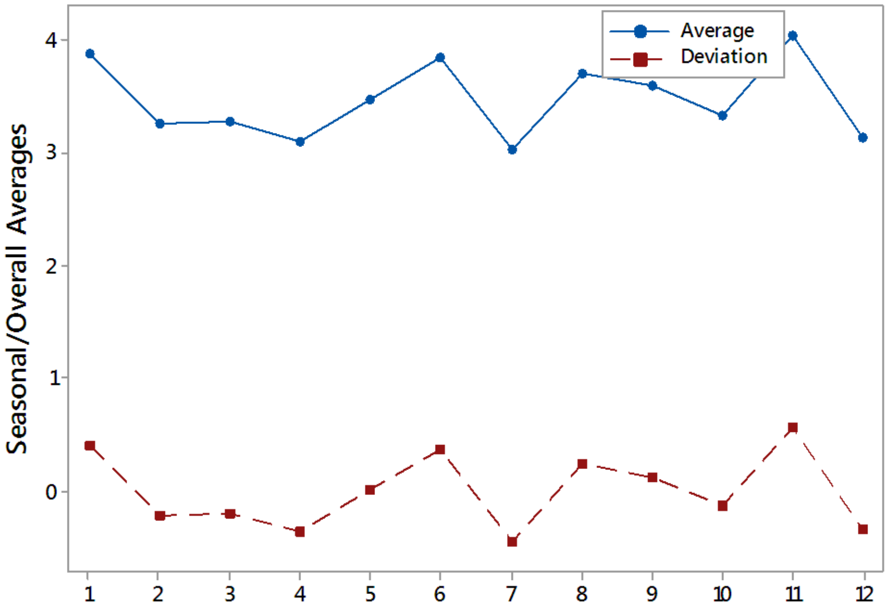

Fig. 3.3. Time plot of seasonal and overall averages

From the transformed series listed in Appendix B, the periodic and seasonal averages in the presence and absence of missing values are listed in Tables 4 and 5. The differences in the periodic and seasonal averages are given in Tables 6 and 7. Also, the differences the periodic and seasonal standard deviations are shown in Tables 8 and 9. The seasonal averages are plotted again the seasonal standard deviation given in figure 3.3 and deviations of seasonal averages from the overall averages shown in figure 3.4. The time plots indicate no significant increase or decrease relatives to any increase or decrease in the seasonal averages, which suggest additive model. The comparison of the standard averages in Table 7 and seasonal standard deviations in Table 9 in the presence and absence of missing values show that, the are approximately same. Therefore, there are no effect in the Buys-Ballot estimates of time series components.

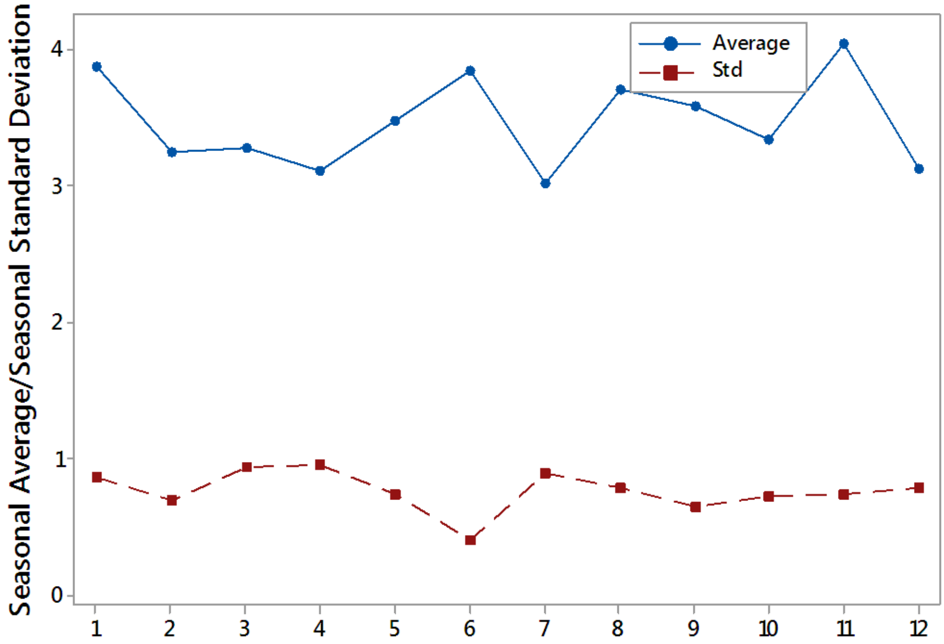

Fig. 3.4 Time plot of seasonal averages and standard deviations

4. CONCLUSION

This study has examined the effect of seasonal averages and seasonal standard deviations on Buys-Ballot estimates of time series components in the presence of missing values. The emphasis is to determine the relationship between seasonal averages and seasonal standard deviations with and without missing values. Results of this analysis show that, 1) the differences between seasonal averages and seasonal standard deviations with and without missing values have insignificant effect, because they are approximated the same. 2) the seasonal averages and standard deviation and the deviations of seasonal averages from the overall averages listed in figures 3.3 and 3.4 indicate no significant increase or decrease relative to any increase or decrease in the seasonal averages, which suggest that, the model for decomposition is additive.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix A. Table for the actual data on number of road traffic accidents with missing values (2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma_{i.}^2$
2013	48	13	40	36	46	48	46	28	30	32	56	56	40.33	138.06
2014	68	14	26	22	38	46	24	30	45	46	14	10	31.92	293.54
2015	88	10	16	30	40	26	46	28	18	12	20	12	28.83	473.79
2016	106	20	4	20	26	44	6	48	58	22	102	86	45.20	1288.30
2017	24	30	40	-	56	56	27	14	30	68	105	19	39.42	777.72
2018	89	39	43	74	56	76	14	112	87	12	60	8	55.83	1126.15
2019	9	54	12	17	18	86	35	134	-	12	63	24	39.70	1480.2
2020	101	86	46	70	58	38	38	68	30	72	-	28	60.08	584.81
2021	22	18	96	8	6	28	4	19	76	38	105	28	37.30	1235.30
$\bar{X}_{.j}$	61.70	32.11	35.89	31.22	38.22	49.78	26.67	53.40	42.89	34.89	67.90	30.11		
$\sigma_{.j}^2$	1368.3	599.61	735.11	631.44	335.44	408.44	256.75	1843.3	668.36	544.11	1208.9	648.11		

Appendix B. Table for the transformed series on number of road traffic accidents with missing values (2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma_{i.}^2$
2013	3.87	3.90	2.89	3.67	3.58	3.82	3.87	3.82	3.33	3.40	4.03	4.03	3.65	0.11
2014	4.22	2.64	3.26	3.09	3.64	3.83	3.17	3.40	3.80	3.81	3.83	2.64	3.32	0.34
2015	4.48	2.30	2.77	3.40	3.69	3.26	3.83	3.33	2.89	2.48	2.99	2.48	3.16	0.41
2016	4.66	2.99	1.39	2.99	3.26	3.78	1.79	3.87	4.06	3.09	4.62	4.45	3.42	1.10
2017	3.18	3.40	3.69	-	4.03	4.03	3.29	2.64	3.40	4.22	4.65	2.94	3.41	0.73
2018	4.49	3.66	3.76	4.30	4.03	4.33	2.64	4.72	2.48	2.48	4.09	2.08	3.76	0.77
2019	2.20	3.99	2.48	2.83	2.89	4.45	3.56	4.90	-	2.48	4.14	3.18	3.30	0.80
2020	4.62	4.45	3.82	4.25	4.06	3.64	3.64	4.22	3.40	4.28	-	3.33	4.01	0.19
2021	3.09	2.89	4.56	2.08	1.79	3.33	1.39	2.94	4.33	3.64	4.65	3.33	3.17	1.11
$\bar{X}_{.j}$	3.87	3.25	3.27	3.10	3.47	3.84	3.02	3.71	3.58	3.33	4.03	3.13		
$\sigma_{.j}^2$	0.74	0.47	0.87	0.89	0.55	0.16	0.80	0.60	0.43	0.53	0.54	0.61		

Appendix C. Table for the actual data on number of road traffic accidents without missing values (2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma_{i.}^2$
2013	48	13	40	36	46	48	46	28	30	32	56	56	40.33	138.06
2014	68	14	26	22	38	46	24	30	45	46	14	10	31.92	293.54
2015	88	10	16	30	40	26	46	28	18	12	20	12	28.83	473.79
2016	106	20	4	20	26	44	6	48	58	22	102	86	45.20	1288.30
2017	24	30	40	35	56	56	27	14	30	68	105	19	39.42	777.72
2018	89	39	43	74	56	76	14	112	87	12	60	8	55.83	1126.15
2019	9	54	12	17	18	86	35	134	85	12	63	24	39.70	1480.2
2020	101	86	46	70	58	38	38	68	30	72	51	28	60.08	584.81
2021	22	18	96	8	6	28	4	19	76	38	105	28	37.30	1235.30
$\bar{X}_{.j}$	61.70	32.11	35.89	31.22	38.22	49.78	26.67	53.40	42.89	34.89	67.90	30.11		
$\sigma_{.j}^2$	1368.3	599.61	735.11	631.44	335.44	408.44	256.75	1843.3	668.36	544.11	1208.9	648.11		

Appendix D. Table for the transformed series on number of road traffic accidents without missing values (2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2013	3.87	3.90	2.89	3.67	3.58	3.82	3.87	3.82	3.33	3.40	4.03	4.03	3.65	0.11
2014	4.22	2.64	3.26	3.09	3.64	3.83	3.17	3.40	3.80	3.81	3.83	2.64	3.32	0.34
2015	4.48	2.30	2.77	3.40	3.69	3.26	3.83	3.33	2.89	2.48	2.99	2.48	3.16	0.41
2016	4.66	2.99	1.39	2.99	3.26	3.78	1.79	3.87	4.06	3.09	4.62	4.45	3.42	1.10
2017	3.18	3.40	3.69	1.38	4.03	4.03	3.29	2.64	3.40	4.22	4.65	2.94	3.41	0.73
2018	4.49	3.66	3.76	4.30	4.03	4.33	2.64	4.72	2.48	2.48	4.09	2.08	3.76	0.77
2019	2.20	3.99	2.48	2.83	2.89	4.45	3.56	4.90	2.48	2.48	4.14	3.18	3.30	0.80
2020	4.62	4.45	3.82	4.25	4.06	3.64	3.64	4.22	3.40	4.28	4.45	3.33	4.01	0.19
2021	3.09	2.89	4.56	2.08	1.79	3.33	1.39	2.94	4.33	3.64	4.65	3.33	3.17	1.11
$\bar{X}_{.j}$	3.87	3.25	3.27	3.10	3.47	3.84	3.02	3.71	3.58	3.33	4.03	3.13		
$\sigma^2_{.j}$	0.74	0.47	0.87	0.89	0.55	0.16	0.80	0.60	0.43	0.53	0.54	0.61		

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