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Research Article

Rational Wave Solutions and Dynamics Properties of the Generalized $(2+1)$-Dimensional Calogero-Bogoyavlenskii-Schiff Equation by Using Bilinear Method

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Through symbolic computation with Maple, fifty-seven sets of rational wave solutions to the generalized Calogero-Bogoyavlenskii-Schiff equation are presented by employing the generalized bilinear operator when the parameter $p=2$. Via the three-dimensional plots and contour plots with the help of Maple, the dynamics of these solutions are described very well. These solutions have greatly enriched the exact solutions of the generalized Calogero-Bogoyavlenskii-Schiff equation on the existing literature. The result will be widely used to describe many nonlinear scientific phenomena.

1. Introduction

It is well known that nonlinear evolution equations (NLEEs) play an important and significant role in describing nonlinear scientific phenomena, such as fluid dynamics, plasma physics, chemistry, marine engineering, optics, and physics. Rational solutions to NLEEs help us understand the physical phenomena they describe in nature. Searching for rational solutions of NLEEs has become a major concern partly due to the availability of computer symbolic systems like Maple, which allow us to deal with some complicated and tedious algebraic calculation. Through the unremitting efforts of mathematicians, many effective methods for solving NLEEs have been established and developed, including Lie symmetry transformation, inverse scattering transformation, Darboux transformation, and Hirota's bilinear theory. Particularly, the Hirota bilinear method was proposed by Hirota, a Japanese scholar, in 1971 to solve the problem of N soliton solution of the nonlinear evolution equation [1]. By using a transformation of the potential function of NLEEs and the definition and properties of the D operator, NLEEs are written in bilinear form, and then, the single-double-multiple soliton solutions of NLEEs can be obtained by using the small parameter expansion method. Based on these methods, one can try to find many interesting analytical solutions of NLEEs, such as the rogue wave solutions [2–4], the multiple wave solutions [5, 6], the lump solutions [7–9], the periodic wave solutions [10–13], the Wronskian solutions [14, 15], the rational solutions [16, 17], the high-order soliton solutions [18, 19], the solitary wave solutions [20, 21], and the other solutions [22–27].

The rest of this paper is arranged as follows. We will get the bilinear form of the gCBS equation in Section 2. In Section 3, the rational wave solutions will be gained by using the polynomial method. A few of conclusion and outlook will be given in Section 4.

2. Bilinear Form of the gCBS Equation

We consider a generalized Calogero-Bogoyavlenskii-Schiff (gCBS) equation [28, 29]:

$$P_{\mathrm{gCBS}}(u,v) := u_t + u_{xxy} + 3uu_y + 3u_x v_y + a u_y + b v_{yy} = 0, \quad (1)$$

where a and b are two constants, or equivalently,

$$v_{tx} + v_{xxxy} + 3v_x v_{xy} + 3v_{xx} v_y + a v_{xy} + b v_{yy} = 0, \tag{2}$$

where $u = v_x$. This is a generalization of a (2 + 1)-dimensional CBS equation

$$v_{tx} + v_{xxxy} + 3v_x v_{xy} + 3v_{xx} v_y = 0, \tag{3}$$

whose coefficients have a different pattern from the original one. The CBS equation was first constructed by Bogoyavlenskii and Schiff in different ways. Bogoyavlenskii used the modified Lax formalism, whereas Schiff derived the same equation by reducing the self-dual Yang-Mills equation. Recently, the lump solutions of the gCBS equation have been solved [28].

Under dependent variable transformation,

$$u = 2(\ln f)_{xx},\ v = 2(\ln f)_x. \tag{4}$$

Equation (1) is transformed into the following generalized bilinear form.

$$B_{\mathrm{gCBS}}(f) := \left(D_{p,t} D_{p,x} + D_{p,x}^3 D_{p,y} + a D_{p,x} D_{p,y} + b D_{p,y}^2\right) f \cdot f, \tag{5}$$

where p being an arbitrarily given natural number [30], often a prime number

$$D_{p,x_1}^{n_1} \cdots D_{p,x_M}^{n_M} a \cdot b(x_1,\cdots,x_M) = \prod_{i=1}^{M} \left(\frac{\partial}{\partial x_i} + \alpha \frac{\partial}{\partial x_i'}\right)^{n_i} a(x_1,\cdots,x_M) b\left(x_1',\cdots,x_M'\right) \times \Big|_{x'=x_1,\cdots,x'=x_M}, \tag{6}$$

where $n_1, \cdots, n_M$ are arbitrary nonnegative integers.

If we take $p = 2$, we obtain the Hirota bilinear equation

$$\begin{aligned} B_{\mathrm{gCBS}}(f) := 2\Big[& f_{tx} f - f_t f_x + f_{xxxy} f - f_{xxx} f_y - 3 f_{xxy} f_x + 3 f_{xx} f_{xy} \\ &+ a\left(f_{xy} f - f_x f_y\right) + b\left(f_{yy} f - f_y^2\right)\Big] = 0. \end{aligned} \tag{7}$$

If we take $p = 3$, we obtained the generalized bilinear equation

$$\begin{aligned} B_{\mathrm{gCBS}}(f) := 2\Big[& f_{tx} f - f_t f_x + 3 f_{xx} f_{xy} + a\left(f_{xy} f - f_x f_y\right) \\ &+ b\left(f_{yy} f - f_y^2\right)\Big] = 0. \end{aligned} \tag{8}$$

The transform (4) is also a characteristic transformation for establishing Bell polynomial theory of soliton equation [31]. Its exact relation is as follows:

$$P_{\mathrm{gCBS}}(u, v) = \left(\frac{B_{\mathrm{gCBS}}(f)}{f^2}\right)_x. \tag{9}$$

Hence, if f solves the generalized bilinear gCBS equation (7), the gCBS equation (1) will be solved.

3. Rational Wave Solutions for the gCBS Equation

In this section, we want to discuss the rational wave solutions to the gCBS equation by using the polynomial solutions.

Let

$$f = \sum_{i=0}^{2} \sum_{j=0}^{2} \sum_{k=0}^{2} C_{i,j,k} x^i y^j t^k, \tag{10}$$

where the $C_{i,j,k}$ are constants, with the help of the computer algebra system Maple, and substituting (10) into equation (7), we obtain a set of algebraic equations. Solving the set of algebraic equations, we can find 57 solutions, but for lack of space, we will list only the following solutions:

Case 1.

$$\begin{aligned} C_{0,0,0} &= \frac{C_{1,0,0}\left(a C_{0,1,1} C_{1,0,0} C_{1,0,1} + b C_{0,1,1}{}^2 C_{1,0,0} + C_{0,0,1} C_{1,0,1}{}^2\right)}{C_{1,0,1}{}^3}, \\ C_{0,0,2} &= -\frac{C_{0,1,1}\left(a C_{1,0,1} + C_{0,1,1} b\right)}{C_{1,0,1}}, \\ C_{0,1,0} &= \frac{C_{1,0,0} C_{0,1,1}}{C_{1,0,1}},\ C_{0,1,2} = C_{0,2,0} = C_{0,2,1} = C_{0,2,2} = C_{1,0,2} \\ &= C_{1,1,0} = C_{1,1,1}, = C_{1,1,2} = C_{1,2,0} = 0, \\ C_{1,2,1} &= C_{1,2,2} = C_{2,0,0} = C_{2,0,1} = C_{2,0,2} = C_{2,1,0} = C_{2,1,1} \\ &= C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \end{aligned} \tag{11}$$

Case 2.

$$\begin{aligned} C_{0,0,0} &= \frac{C_{1,0,0} C_{0,0,2}}{C_{1,0,2}},\ C_{0,0,1} = \frac{C_{1,0,1} C_{0,0,2}}{C_{1,0,2}},\ C_{0,1,0} = -\frac{a C_{1,0,0}}{b}, \\ C_{0,1,1} &= -\frac{a C_{1,0,1}}{b},\ C_{0,1,2} = -\frac{a C_{1,0,2}}{b},\ C_{0,2,0} = 0, \\ C_{0,2,1} &= C_{0,2,2} = C_{1,1,0} = C_{1,1,1} = C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} \\ &= C_{2,0,0} = C_{2,0,1} = C_{2,0,2} = C_{2,1,0} = 0, \\ C_{2,1,1} &= C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \end{aligned} \tag{12}$$

Case 3.

$$\begin{aligned} C_{0,0,0} &= C_{0,1,0} = 0,\ C_{0,1,2} = \frac{C_{0,0,2} C_{0,1,1}}{C_{0,0,1}},\ C_{0,2,0} = C_{0,2,1} = C_{0,2,2} \\ &= C_{1,0,0} = 0,\ C_{1,0,1} = -\frac{b C_{0,1,1}}{a}, \end{aligned}$$

$$C_{1,0,2} = -\frac{bC_{0,1,1}C_{0,0,2}}{C_{0,0,1}a}, C_{1,1,0} = C_{1,1,1} = C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = C_{2,0,0} = C_{2,0,1} = 0,$$

$$C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \quad (13)$$

Case 4.

$$C_{0,0,1} = C_{0,1,1} = 0, C_{0,1,2} = \frac{C_{0,1,0}C_{0,0,2}}{C_{0,0,0}}, C_{0,2,0} = C_{0,2,1} = C_{0,2,2} = 0, C_{1,0,0} = -\frac{bC_{0,1,0}}{a}, C_{1,0,1} = 0,$$

$$C_{1,0,2} = -\frac{bC_{0,1,0}C_{0,0,2}}{aC_{0,0,0}}, C_{1,1,0} = C_{1,1,1} = C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = C_{2,0,0} = C_{2,0,1} = 0,$$

$$C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \quad (14)$$

Case 5.

$$C_{0,0,2} = 0, C_{0,1,1} = \frac{C_{0,0,1}C_{0,1,0}}{C_{0,0,0}}, C_{0,1,2} = C_{0,2,0} = C_{0,2,1} = C_{0,2,2} = 0, C_{1,0,0} = -\frac{bC_{0,1,0}}{a}, C_{1,0,2} = C_{1,1,0} = 0,$$

$$C_{1,0,1} = -\frac{C_{0,0,1}C_{0,1,0}b}{aC_{0,0,0}}, C_{1,1,1} = C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = C_{2,0,0} = C_{2,0,1} = C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = 0,$$

$$C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \quad (15)$$

Case 6.

$$C_{0,0,0} = \frac{C_{0,1,0}{}^2C_{2,0,0} + C_{0,2,0}C_{1,0,0}{}^2}{4C_{0,2,0}C_{2,0,0}}, \quad C_{0,0,1} = -\frac{aC_{0,1,0}C_{2,0,0} - bC_{0,2,0}C_{1,0,0}}{C_{2,0,0}}, C_{0,1,1} = -2C_{0,2,0}a,$$

$$C_{0,0,2} = \frac{C_{0,2,0}\left(a^2C_{2,0,0} + b^2C_{0,2,0}\right)}{C_{2,0,0}}, C_{0,1,2} = C_{0,2,1} = C_{0,2,2} = 0, C_{1,0,1} = 2bC_{0,2,0,}$$

$$C_{1,2,1} = C_{1,2,2} = C_{2,0,1} = C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0,$$

$$C_{1,0,2} = C_{1,1,0} = C_{1,1,1} = C_{1,1,2} = C_{1,2,0} = 0. \quad (16)$$

Case 7.

$$C_{0,0,1} = \frac{a^2C_{1,0,1}{}^2 - b^2C_{0,1,1}{}^2}{4a^2C_{2,0,1}}, C_{0,0,2} = -\frac{a\left(aC_{1,0,1} + bC_{0,1,1}\right)}{b}, \quad C_{0,1,2} = \frac{2a^3C_{2,0,1}}{b^2}, C_{0,2,1} = -\frac{a^2C_{2,0,1}}{b^2},$$

$$C_{0,0,0} = C_{0,1,0} = C_{0,2,0} = 0, C_{0,2,2} = 0, C_{1,0,0} = 0, C_{1,0,2} = -\frac{2a^2C_{2,0,1}}{b}, C_{1,1,0} = C_{1,1,1} = C_{1,1,2} = 0,$$

$$C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = C_{2,0,0} = C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \quad (17)$$

Case 8.

$$C_{0,0,0} = C_{0,1,0} = C_{0,1,2} = C_{0,2,0} = C_{0,2,1} = C_{0,2,2} = C_{1,0,0} = 0, C_{1,0,1} = -\frac{C_{0,1,1}{}^2b}{C_{0,1,1}a + C_{0,0,2}}, C_{1,0,2} = 0,$$

$$C_{1,1,0} = C_{1,1,1} = C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = C_{2,0,0} = C_{2,0,1} = C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = C_{2,1,2} = 0,$$

$$C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \quad (18)$$

Case 9.

$$C_{0,0,2} = C_{0,1,1} = C_{0,1,2} = C_{0,2,0} = C_{0,2,1} = C_{0,2,2} = 0, C_{1,0,1} = \frac{C_{1,1,0}C_{0,0,1}}{C_{0,1,0}}, C_{1,0,2} = C_{1,1,1} = 0,$$

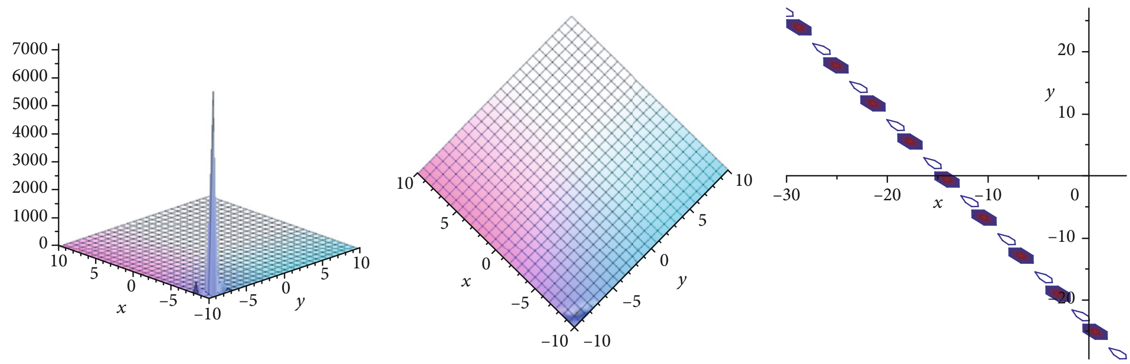
$$C_{1,0,0} = \frac{a^2C_{0,0,0}C_{0,1,0}{}^2C_{1,1,0} - abC_{0,1,0}{}^4 + 2aC_{0,0,0}C_{0,0,1}C_{0,1,0}C_{1,1,0} - bC_{0,0,1}C_{0,1,0}{}^3}{C_{0,1,0}\left(a^2C_{0,1,0}{}^2 + 2aC_{0,0,1}C_{0,1,0} + C_{0,0,1}{}^2\right)} - \frac{6bC_{0,1,0}{}^2C_{1,1,0}{}^2 + C_{0,0,0}C_{0,0,1}{}^2C_{1,1,0}}{C_{0,1,0}\left(a^2C_{0,1,0}{}^2 + 2aC_{0,0,1}C_{0,1,0} + C_{0,0,1}{}^2\right)},$$

$$C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = 0, C_{2,0,0} = -\frac{bC_{0,1,0}C_{1,1,0}}{C_{0,1,0}a + C_{0,0,1}}, C_{2,0,1} = C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = 0, C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0. \quad (19)$$

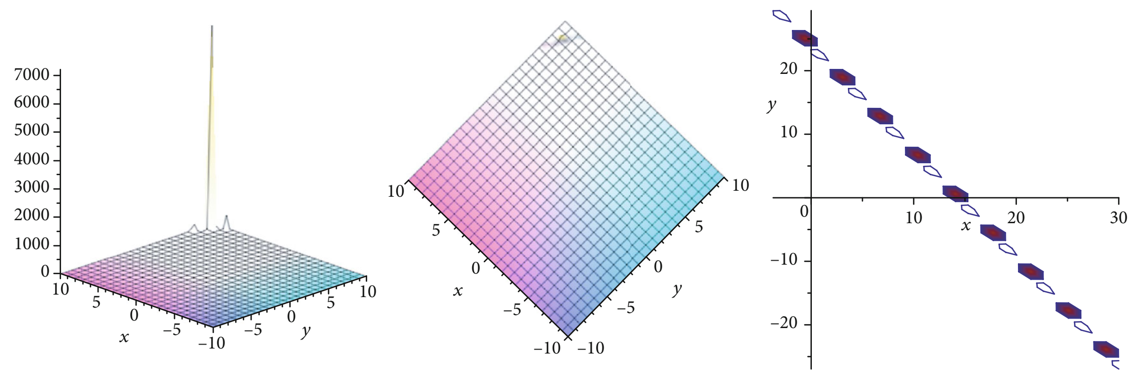
Case 10.

$$C_{0,0,0} = \frac{C_{1,1,0}\left(a^2C_{0,1,1}C_{1,0,1} + abC_{0,1,1}{}^2 + 6bC_{1,1,1}{}^2\right)}{a^2C_{1,1,1}{}^2}, \quad C_{0,0,1} = \frac{a^2C_{0,1,1}C_{1,0,1} + abC_{0,1,1}{}^2 + 6bC_{1,1,1}{}^2}{a^2C_{1,1,1}},$$

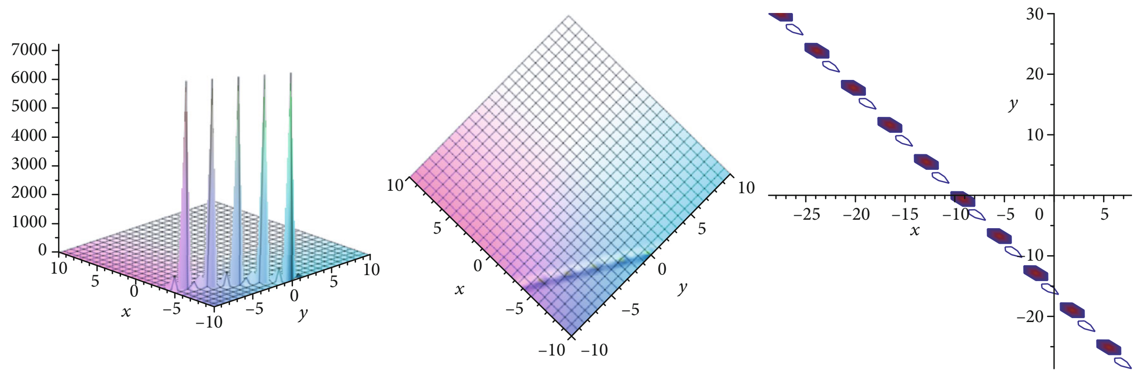
$$C_{0,0,2} = 0, C_{0,1,0} = \frac{C_{1,1,0}C_{0,1,1}}{C_{1,1,1}}, C_{0,1,2} = C_{0,2,0} = C_{0,2,1} = C_{0,2,2} = 0, C_{1,0,0} = \frac{C_{1,1,0}C_{1,0,1}}{C_{1,1,1}}, C_{1,0,2} = 0,$$



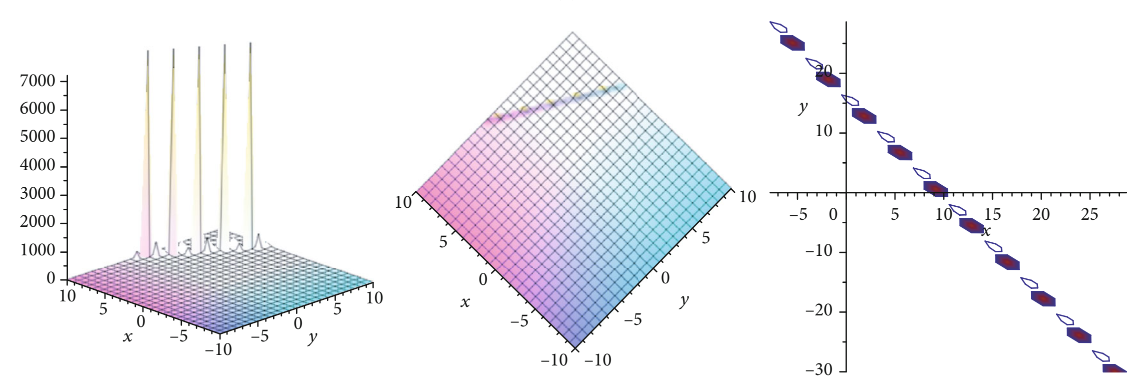
(a)



(b)

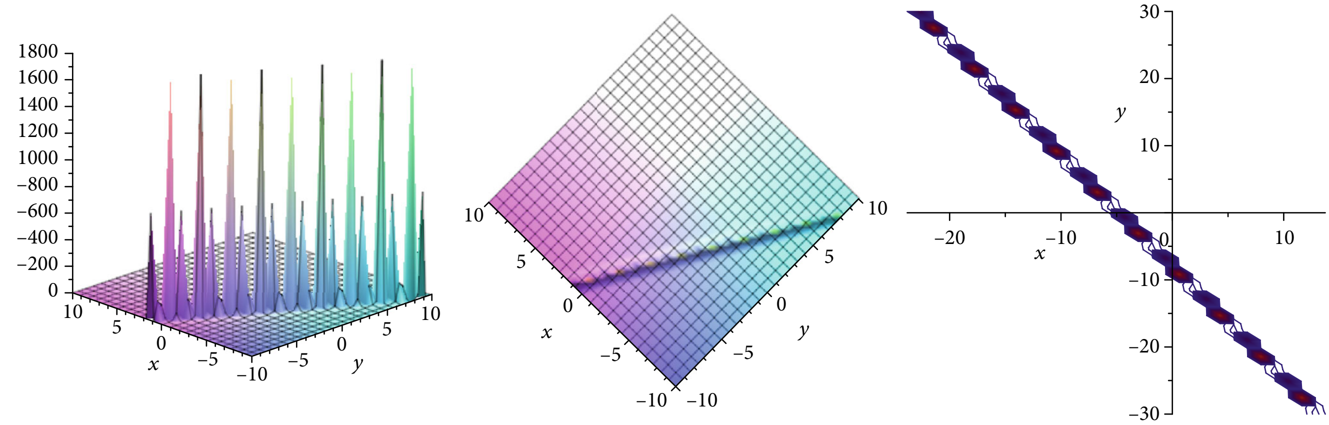


(c)

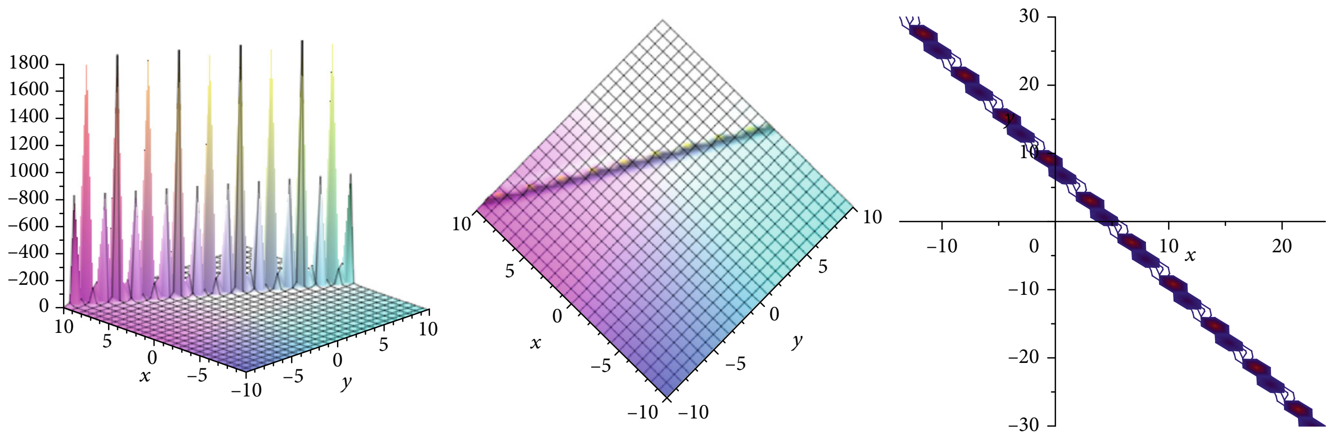


(d)

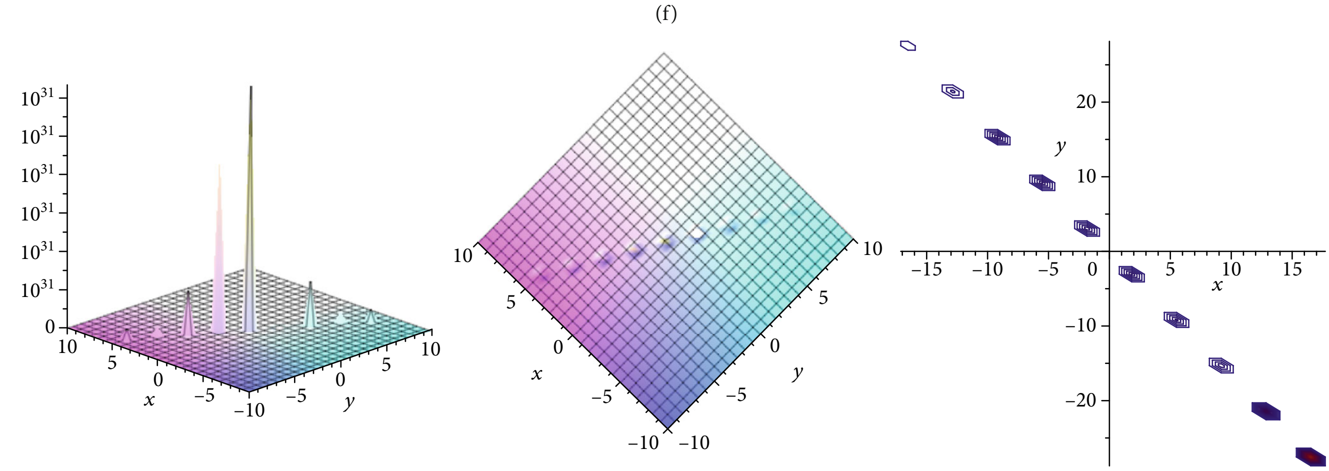
Figure 1: Continued.



(e)



(f)



(g)

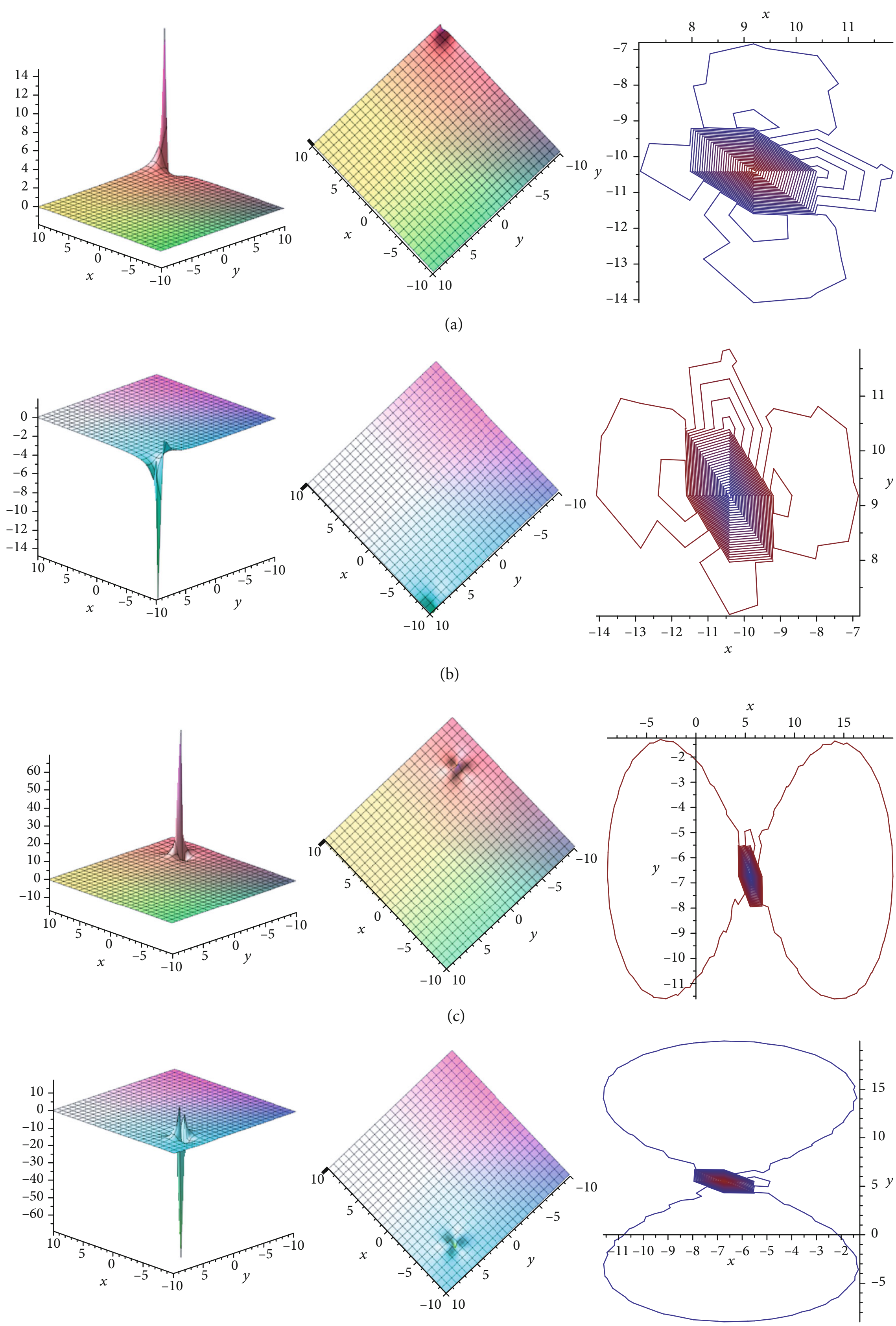
FIGURE 1: 3D plots and contour plots of equation (21) by choosing $a = 7$, $b = 5$, $C_{0,0,1} = 6$, $C_{0,1,1} = 6$, $C_{1,0,0} = 8$, $C_{1,0,1} = 10$.

$$
\begin{aligned}
& C_{1,1,2} = C_{1,2,0} = C_{1,2,1} = C_{1,2,2} = 0,\ C_{2,0,0} = -\frac{bC_{1,1,0}}{a},\ C_{2,0,1} \\
& \quad = -\frac{bC_{1,1,1}}{a},\ C_{2,0,2} = C_{2,1,0} = C_{2,1,1} = 0, \\
& C_{2,1,2} = C_{2,2,0} = C_{2,2,1} = C_{2,2,2} = 0.
\end{aligned} \tag{20}
$$

The constant $C_{i,j,k}$ involved are arbitrary as long as the solution is meaningful, considering the transformation of the coefficient $C_{i,j,k}$.

By observation, we can divide the 57 sets of solutions into two classes, Cases 1–5 belong to the first class, Cases 6–10 belong to the second class.

Here, let us first discuss the solution of Case 1. Through the transformation (4), we get the following solution to the gCBS equation



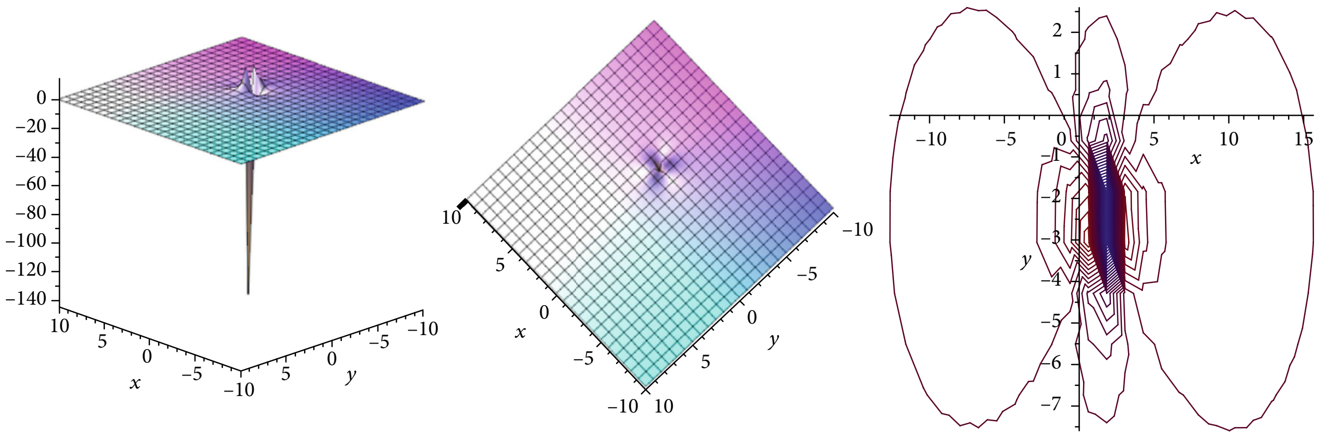
(a)

(b)

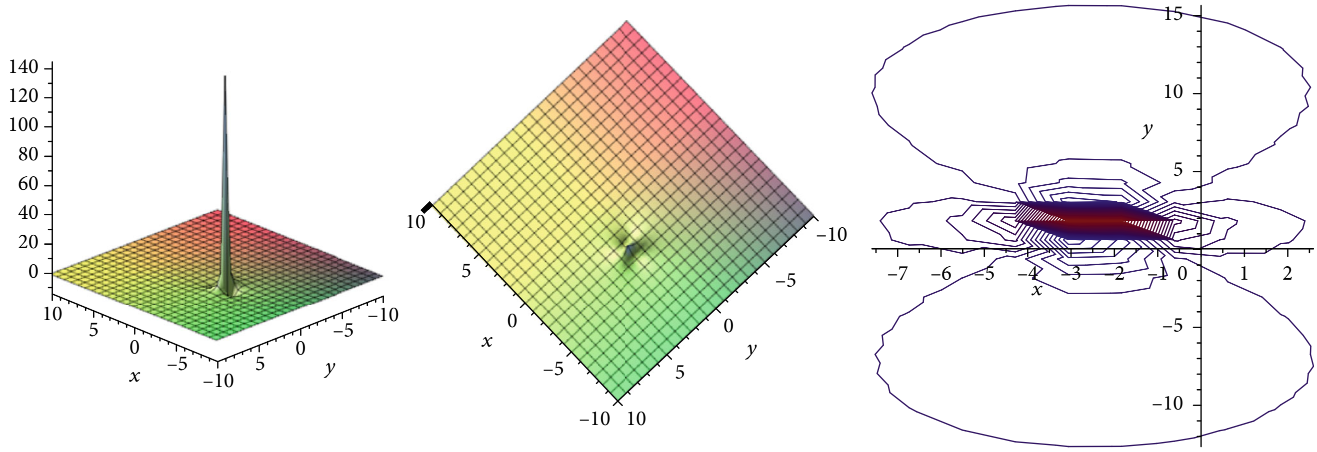
(c)

(d)

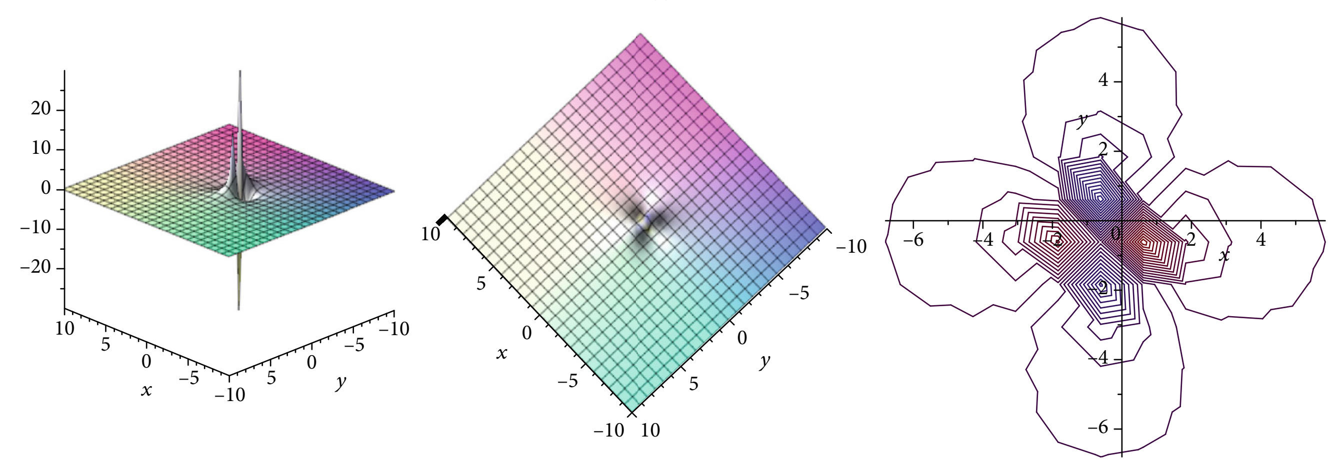
Figure 2: Continued.



(e)



(f)



(g)

Figure 2: 3D plots and contour plots of equation (24) by choosing $a = 2$, $b = 2$, $C_{0,1,0} = 1$, $C_{0,2,0} = 1$, $C_{1,0,0} = 1$, $C_{2,0,0} = 1$.

$$u = -2\frac{p^2}{f^2}, \tag{21}$$

where the functions p and f are given as follows:

$$\begin{aligned} p &= C_{1,0,1}t + C_{1,0,0}, \\ f &= \frac{C_{1,0,0}\left(aC_{0,1,1}C_{1,0,0}C_{1,0,1} + bC_{0,1,1}{}^2C_{1,0,0} + C_{0,0,1}C_{1,0,1}{}^2\right)}{C_{1,0,1}{}^3} \\ &\quad + C_{0,0,1}t - \frac{C_{0,1,1}\left(aC_{1,0,1} + bC_{0,1,1}\right)t^2}{C_{1,0,1}} + \frac{C_{1,0,0}C_{0,1,1}y}{C_{1,0,1}} \\ &\quad + C_{0,1,1}yt + C_{1,0,1}xt + C_{1,0,0}x. \end{aligned} \tag{22}$$

In order to analyze the dynamics properties briefly, we would like to discuss the evolution characteristic. By choosing appropriate values of these parameters in (21), we set

$$a = 7, b = 5, C_{0,0,1} = 6, C_{0,1,1} = 6, C_{1,0,0} = 8, C_{1,0,1} = 10. \tag{23}$$

The three-dimensional dynamic graphs and corresponding contour plots of the solution were successfully depicted in Figure 1.

Correspondingly, Figure 1 displays the whole process of the creation and disappearance of the rogue wave aroused by solution (21). From the coordinates, that rogue wave is almost uniform over time. When $t = -1.5$ in Figure 1(a), the wave appears gradually, and this obvious wave is moving diagonally toward the center along the x- and y-axes (Figures 1(a), 1(c), and 1(e)). When $t = 0.9$ in Figure 1(g)), it comes to the center. Then, this wave is moving diagonally away from the center along the x- and y-axes (Figures 1(f), 1(d), and 1(b)). When $t = 3.3$ in Figure 1(g), this obvious wave disappears gradually. As can be seen, when time approaches zero, the amplitude of rogue wave increases the maximum and then decreases. So the asymptotic behavior of u can be obtained, the solution $u \longrightarrow 0$ as $t \longrightarrow \infty$. We hope our work in this paper contributes to the study of multidimensional and higher-order rogue waves.

Second, let us discuss the solution of Case 6, and the rest of the cases are the same, the rational wave solutions to the gCBS equation read

$$u = 4\frac{C_{2,0,0}}{f} - 2\frac{p^2}{f^2}, \tag{24}$$

where the functions p and f are given as follows:

$$\begin{aligned} p &= 2btC_{0,2,0} + 2xC_{2,0,0} + C_{1,0,0}, \\ f &= \frac{C_{0,1,0}{}^2C_{2,0,0} + C_{0,2,0}C_{1,0,0}{}^2}{4C_{0,2,0}C_{2,0,0}} - \frac{\left(aC_{0,1,0}C_{2,0,0} - bC_{0,2,0}C_{1,0,0}\right)t}{C_{2,0,0}} \\ &\quad + \frac{C_{0,2,0}\left(a^2C_{2,0,0} + b^2C_{0,2,0}\right)t^2}{C_{2,0,0}} + C_{0,1,0}y - 2C_{0,2,0}ayt + C_{0,2,0}y^2 \\ &\quad + 2bC_{0,2,0}xt + C_{1,0,0}x + C_{2,0,0}x^2. \end{aligned} \tag{25}$$

In order to analyze the dynamics properties briefly, we would like to discuss the evolution characteristic. By choosing appropriate values of these parameters in (24), we set

$$a = 2, b = 2, C_{0,1,0} = 1, C_{0,2,0} = 1, C_{1,0,0} = 1, C_{2,0,0} = 1. \tag{26}$$

The three-dimensional dynamic graphs of the solution and corresponding contour plots were successfully depicted in Figure 2. Correspondingly, Figure 2 displays the whole motion process of the rogue wave aroused by solution (24). As can be seen in Figure 2, when $t = -5$ in Figure 2(a), an obvious wave starts to appear, and this obvious wave is moving toward the center along the negative y-axis (Figures 2(a), 2(c), and 2(e)). When $t = 0$ in Figure 2(g), it comes to the center. Then, this wave is moving away from the center along the positive y-axis (Figures 2(f), 2(d), and 2(b)). When $t = 5$ in Figure 2(b), this obvious wave reaches the edge and is about to disappear. From the coordinates, the rational wave is almost uniform over time. When the sign of the t is reversed, we can receive that the waves are centrosymmetric with respect to coordinates' origin, and the waves have the same amplitude.

4. Conclusion and Outlook

In conclusion, based on the generalized bilinear form of the gCBS equation, we obtained rich rational wave solutions of the gCBS equation. We obtained fifty-seven sets of rational wave solutions and list two solutions to analyze. We successfully depicted the three-dimensional graphs and the corresponding contour plots of the rational wave solutions. The 3D plots and contour plots in Figures 1 and 2 are given to display the dynamic process of the rogue wave, and the rational wave solutions contribute to the study of multidimensional and higher-order rogue waves. Some researchers have studied the exact solutions of $(2 + 1)$-dimensional Calogero-Bogoyavlenskii-Schiff equation, such as lump solutions [28], breather wave solutions, and periodic lump soliton solutions [29]. The obtained new rational wave solutions in this paper are different from the existing solutions. These solutions will greatly expand the exact solutions of $(2 + 1)$-dimensional Calogero-Bogoyavlenskii-Schiff equation on the existing literature [28, 29]. It is worthwhile to mention that the proposed method is reliable and effective and gives more solutions. We can imagine that the bilinear form of the gCBS equation and such structural solutions will be useful to investigate many nonlinear dynamics of interaction phenomena in fluids and plasmas fields.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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