Journal of Scientific Research & Reports
2(2): 571-581, 2013; Article no. JSRR.2013.008

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A Generalized Sub-ODE Method and Applications for Nonlinear Evolution Equations

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Authors' contributions

This work was carried out in collaboration between all authors. Author FX managed the analyses of the study. All authors read and approved the final manuscript.

Research Article

Received 16th June 2013
Accepted 15th August 2013
Published 22nd August 2013

ABSTRACT

In this paper, a generalized Bernoulli sub-ODE method is applied to seek exact solutions for nonlinear evolution equations. This method is based on the homogeneous principle, and is effective in seeking new travelling wave solutions. As applications, we apply this method to solve (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili (BKP) equation, and with the aid of mathematical software, some new exact travelling wave solutions for this equation are found.

Keywords: Bernoulli sub-ODE method; travelling wave solutions; (2+1) dimensional BKP equation; nonlinear evolution equation.

1. INTRODUCTION

Recently searching for exact travelling wave solutions of nonlinear evolution equations (NLEEs) has gained more and more popularity, and many effective methods have been presented so far. Some of these approaches are the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6-8], the non-linear transform method [9], the inverse scattering transform [10], the Backlund transform [11,12], the Hirota's bilinear method [13,14], the generalized Riccati equation method [15,16] and so on.

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Among the investigations for seeking exact solutions nonlinear equations, Prof. Wen-Xiu Ma has done much pioneer work [17-21]. In [17], Prof. Wen-Xiu Ma presented one combined ansätze method, the idea of which is to make the unknown variable u to be a practicable function g(v) of the ansätze unknown variable v, which satisfies a differential equation solvable by quadratures. The crucial point of this method is to choose the proper ansätze equations solvable by quadratures. The Bernoulli equation was also listed there as one useful ansätze equation, and the general solutions of the Bernoulli equation was also presented. Based on this method, some explicit traveling wave solutions to a Kolmogorov Petrovskii Piskunov equation were presented. It is worthy to note that the extended tanh function method and the G'/G-expansion method are both special cases of the method in [17]. In [18], a transformed rational function method was proposed and applied to seek exact solutions of (3+1)-dimensional Jimbo-Miwa equation. In [19], A multiple exp-function method to exact multiple wave solutions of nonlinear partial differential equations was proposed. Then in [20], the multiple exp-function method was used to construct three-wave solutions to the (3+1)-dimensional generalized KP and BKP equations. In [21], Frobenius integrable decompositions were introduced for partial differential equations.

Motivated by the ideas in [17], in this paper, we apply the Bernoulli sub-ODE method [22, 23] to construct exact travelling wave solutions for NLEEs. Firstly, we reduce the NLEEs to ODEs by a travelling wave variable transformation. Secondly, we suppose the solution can be expressed in an polynomial in a variable G, where $G=G(\xi)$ satisfied the Bernoulli equation. Thirdly, the degree of the polynomial can be determined by the homogeneous balance method, and the coefficients can be obtained by solving a set of algebraic equations.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding travelling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to find exact travelling wave solutions of (2+1) dimensional BKP equation. In the last Section, some conclusions are presented.

2. DESCRIPTION OF THE BERNOULLI SUB-ODE METHOD

In this section we give the main steps for Bernoulli Sub-ODE Method to seek exact solutions for nonlinear evolution equations.

Consider the following ordinary differential equation (ODE):

$$G'+\lambda G=\mu G^2 \tag{2.1}$$

where $\lambda\neq 0, G=G(\xi)$.

When $\mu\neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G=\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}} \tag{2.2}$$

where d is an arbitrary constant.

When $\mu = 0$, the solution of Eq. (2.1) is denoted by

$$G = de^{-\lambda\xi} \tag{2.3}$$

Suppose that a nonlinear equation, say in two or three independent variables x, y and t, is given by

$$P(u, u_{t,} u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} ......) = 0 \tag{2.4}$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 1. We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.5}$$

The travelling wave variable (2.5) permits us reducing Eq. (2.4) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.6}$$

Step 2. Suppose that the solution of (2.6) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.7}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and non-linear terms appearing in (2.6).

Step 3. Substituting (2.7) into (2.6) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.6) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the travelling wave solutions of the nonlinear evolution equation (2.6).

In the following, we will apply the method described above to seek exact travelling wave solutions for the (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation.

3. APPLICATION OF THE BERNOULLI SUB-ODE METHOD FOR (2+1) DIMENSIONAL BOUSSINESQ AND KADOMTSEV-PETVIASHVILI EQUATION

Consider the (2+1) dimensional BKP equation [24,25]:

$$u_y = q_x \tag{3.1}$$

$$v_x = q_y \tag{3.2}$$

$$q_t = q_{xxx} + q_{yyy} + 6(qu)_x + 6(qv)_y \tag{3.3}$$

In order to obtain the travelling wave solutions of (3.1), (3.2) and (3.3), similar to the section 3, we suppose that

$$u(x,y,t) = u(\xi), v(x,y,t) = v(\xi), q(x,y,t) = q(\xi), \xi = ax + dy - ct \tag{3.4}$$

where a, d, c are constants that to be determined later.

By using the wave variable (3.4), Eq. (3.1)-(3.3) can be converted into ODEs:

$$du' - aq' = 0 \tag{3.5}$$

$$av' - dq' = 0 \tag{3.6}$$

$$(a^3 + d^3)q''' - cq' - 6auq' - 6aqu' - 6dvq' - 6dqv' = 0 \tag{3.7}$$

Integrating the ODEs above, we obtain

$$du - aq = g_1 \tag{3.8}$$

$$av - dq = g_2 \tag{3.9}$$

$$(a^3 + d^3)q'' - cq - 6auq - 6dvq = g_3 \tag{3.10}$$

Suppose that the solution of (10) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{l} a_i G^i \tag{3.11}$$

$$v(\xi) = \sum_{i=0}^{m} b_i G^i \tag{3.12}$$

$$q(\xi)=\sum_{i=0}^{n} c_i G^i \tag{3.13}$$

where a_i, b_i, c_i are constants and $G=G(\xi)$ satisfies Eq. (2.1).

Balancing the order of u' and q' in Eq. (3.8), the order of v' and q' in Eq. (3.9) and the order of q''' and vq' in Eq. (3.10), we have

$l+1=n+1, m+1=n+1, n+3=m+n+1 \Rightarrow l=m=n=2$. So Eq. (3.11)-(3.13) can be rewritten as:

$$u(\xi)=a_2G^2+a_1G+a_0, a_2\neq 0 \tag{3.14}$$

$$v(\xi)=b_2G^2+b_1G+b_0, b_2\neq 0 \tag{3.15}$$

$$q(\xi)=c_2G^2+c_1G+c_0, c_2\neq 0 \tag{3.16}$$

where a_i, b_i, c_i are constants to be determined later.

Substituting Eq. (3.14)-(3.16) into the ODEs (3.8)-(3.10), collecting all terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq. (3.8)

$$G^0: a_0d-ac_0-g_1=0$$

$$G^1: a_1d-ac_1=0$$

$$G^2: a_2d-ac_2=0$$

For Eq. (3.9)

$$G^0: ab_0-g_2-dc_0=0$$

$$G^1: ab_1-dc_1=0$$

$$G^2: -dc_2+ab_2=0$$

For Eq. (3.10)

$$G^0 : -cc_0 - g_3 - 6db_0c_0 - 6aa_0c_0 = 0$$

$$G^1 : -6aa_1c_0 - 6db_1c_0 - 6db_0c_1 + c_1\lambda^2(a^3 + d^3) - 6aa_0c_1 = 0$$

$$G^2 : -6aa_1c_1 - 6db_1c_1 - 6db_0c_2 - 6db_2c_0 + 4c_2\lambda^2(a^3 + d^3) - cc_2$$
$$-3c_1\mu\lambda(a^3 + d^3) - 6aa_2c_0 = 0$$

$$G^3 : -6aa_2c_1 - 6db_2c_1 - 6db_1c_2 - 6aa_1c_2 + 2c_1\mu^2(a^3 + d^3) - 10c_2\mu\lambda(a^3 + d^3) = 0$$

$$G^4 : -6aa_2c_2 - 6db_2c_2 + 6c_2\mu^2(a^3 + d^3) = 0$$

Solving the algebraic equations above with the mathematical software Maple, yields:

Case 1:

$$a_0 = a_0, a_1 = -\mu\lambda a^2, a_2 = a^2\mu^2, b_0 = b_0, b_1 = -\mu\lambda d^2, b_2 = d^2\mu^2, a = a,$$
$$c_0 = c_0, c_1 = -\mu\lambda da, c_2 = d\mu^2 a, g_2 = ab_0 - dc_0, g_1 = da_0 - ac_0, d = d,$$
$$c = \frac{-6a^3c_0 - 6d^3c_0 + a^4d\lambda^2 - 6d^2b_0a - 6a^2a_0d + d^4\lambda^2 a}{ad}$$
$$g = -c_0\frac{-6a^3c_0 - 6d^3c_0 + a^4d\lambda^2 + d^4\lambda^2 a}{ad} \tag{3.17}$$

where a_0, b_0, c_0, a, d are arbitrary constants.

Assume $\mu \neq 0$, then substituting the results above into (3.14)-(3.16), combining with (2.2) we can obtain the travelling wave solution of (2+1) dimensional BKP equation as follows:

$$u_1(\xi) = a^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 - \mu\lambda a^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + a_0 \tag{3.18}$$

$$v_1(\xi) = d^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 - \mu\lambda d^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + b_0 \tag{3.19}$$

$$q_1(\xi) = d\mu^2 a\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 - \mu\lambda da\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + c_0 \tag{3.20}$$

$$\xi = ax + dy - \frac{-6a^3c_0 - 6d^3c_0 + a^4d\lambda^2 - 6d^2b_0a - 6a^2a_0d + d^4\lambda^2a}{ad}t \quad (3.21)$$

Case 2:

$$a_0 = a_0, a_1 = a_1, a_2 = d^2\mu^2, b_0 = b_0, b_1 = a_1, b_2 = d^2\mu^2, d = d, c = 6d(-b_0 + a_0),$$
$$c_0 = c_0, c_1 = -a_1, c_2 = -d^2\mu^2, g_2 = -db_0 - dc_0, g_1 = da_0 + dc_0, a = -d, g_3 = 0, \quad (3.22)$$

where a_0, b_0, c_0, a_1, d are arbitrary constants.

Similarly under the condition $\mu \neq 0$, we can obtain another travelling wave solution of (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation as follows:

$$u_2(\xi) = d^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 + a_1\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + a_0 \quad (3.23)$$

$$v_2(\xi) = d^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 + a_1\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + b_0 \quad (3.24)$$

$$q_2(\xi) = -d^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 - a_1\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + c_0 \quad (3.25)$$

$$\xi = ax + dy - 6d(-b_0 + a_0)t \quad (3.26)$$

Case 3:

$$a_0 = a_0, a_1 = \frac{1}{2}d^2\mu\lambda\left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right), a_2 = d^2\mu^2\left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right), b_0 = b_0,$$
$$b_1 = \frac{1}{2}d^2\mu\lambda, b_2 = d^2\mu^2, c_0 = c_0, c_1 = \frac{1}{2}d^2\mu\lambda\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right), c_2 = d^2\mu^2\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)$$
$$d = d, a = \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)d, c = -6da_0\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right) - 6db_0, g_3 = 0,$$
$$g_1 = da_0 - dc_0\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right), g_2 = db_0\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right) - dc_0, \quad (3.27)$$

where a_0, b_0, c_0, d are arbitrary constants. Then

$$u_3(\xi)=d^2\mu^2(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})^2+\frac{1}{2}d^2\mu\lambda(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+a_0 \quad (3.28)$$

$$v_3(\xi)=d^2\mu^2(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})^2+\frac{1}{2}d^2\mu\lambda(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+b_0 \quad (3.29)$$

$$q_3(\xi)=d^2\mu^2(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})^2+\frac{1}{2}d^2\mu\lambda(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+c_0 \quad (3.30)$$

$$\xi=(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)dx+dy+[6da_0(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)+6db_0]t \quad (3.31)$$

where $\mu\neq 0$.

Case 4:

$$a_0=a_0,a_1=\frac{1}{2}d^2\mu\lambda(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i),a_2=d^2\mu^2(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i),b_0=b_0,$$
$$b_1=-a_1(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i),b_2=d^2\mu^2,c_0=c_0,c_1=-a_1(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i),c_2=d^2\mu^2(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)$$
$$d=d,a=(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)d,c=-6da_0(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)-6db_0,g_3=0,$$
$$g_1=da_0-dc_0(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i),g_2=db_0(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)-dc_0, \quad (3.32)$$

where a_0,b_0,c_0,d are arbitrary constants. Then

$$u_4(\xi)=d^2\mu^2(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})^2+\frac{1}{2}d^2\mu\lambda(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+a_0 \quad (3.33)$$

$$v_4(\xi)=d^2\mu^2(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})^2-a_1(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+b_0 \quad (3.34)$$

$$q_3(\xi)=d^2\mu^2(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})^2-a_1(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+c_0 \quad (3.35)$$

$$\xi = (\frac{1}{2} \pm \frac{\sqrt{3}}{2} i)dx + dy + [6da_0(\frac{1}{2} \pm \frac{\sqrt{3}}{2} i) + 6db_0]t \quad (3.36)$$

where $\mu \neq 0$.

Remark 1: In [24], Zheng presented some exact solutions including hyperbolic function solutions, trigonometric function solutions, rational function solutions, and some soliton solutions for Eqs. (3.1)-(3.3). We note that our solutions (3.18)-(3.20), (3.23)-(3.25) are expressed in the Exp function, which are different from Zheng's results. Furthermore, our solutions (3.28)-(3.30), (3.33)-(3.35) are complex solutions, which are essentially different from the solutions proposed in [24].

Remark 2: The travelling wave solutions mentioned above have not been reported by other authors to our best knowledge.

4. CONCLUSIONS

In this paper, some new travelling wave solutions of (2+1) dimensional BKP equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the travelling wave variable as well as integrating can be expressed by an $m-$th degree polynomial in G, where $G = G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. Also this method can be used to many other nonlinear problems.

CONSENT

All authors declare that 'written informed consent was obtained from the patient (or other approved parties) for publication of this case report and accompanying images.

ACKNOWLEDGEMENTS

The authors would like to thank the reviewers very much for their valuable suggestions on improving this paper.

COMPETING INTERESTS

Authors have declared that no competing interests exist

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