academicJournals

Vol. 7(1), pp. 1-6, February, 2014
DOI: 10.5897/AJMCSR2014.0532
ISSN 2006-9731 ©2014 Academic Journals
http://www.academicjournals.org/AJMCSR

African Journal of Mathematics and Computer Science Research

Full Length Research Paper

Application of exp $(-\phi(\xi))$- expansion method to find the exact solutions of Shorma-Tasso-Olver Equation

Nizhum Rahman[1], Md. Nur Alam[1], Harun-Or-Roshid[1]*, Selina Akter[1] and M. Ali Akbar[2]

[1]Department of Mathematics, Pabna University of Science and Technology, Bangladesh.
[2]Department of Applied Mathematics, University of Rajshahi, Bangladesh.

Accepted 22 January, 2014

In this work, we present traveling wave solutions for the Shorma-Tasso-Olver equation. The idea of exp $(-\phi(\xi))$- expansion method is used to obtain exact solutions of that equation. The traveling wave solutions are expressed by the exponential functions, the hyperbolic functions, the trigonometric functions solutions and the rational functions. It is shown that the method is awfully effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical science and engineering.

Key words: The exp $(-\phi(\xi))$- expansion method, the Shorma-Tasso-Olver equation, nonlinear partial differential equation, homogeneous balance, traveling wave solutions, solitary wave solutions.

INTRODUCTION

The study of nonlinear evolution equation (NLEE) has much remarkable progress in the past few decades. Most of the phenomena in real world can be described using non-linear equations. A nonlinear phenomenon plays a vital role in applied mathematics, physics and engineering branches. Most of the complex nonlinear equations in plasma physics, fluid dynamics, chemistry, biology, mechanics, elastic media and optical fibers etc., can be explained by nonlinear evolution equations. There are a lot of NLEEs that are integrated using various mathematical techniques.

In recent times, many powerful and effective methods have been presented such as the Ansatz method (Hu, 2001a,b; Wang et al., 2013), the Complex hyperbolic function method (Zayed et al., 2006; Chow, 1995), the (G'/G)-expansion method (Wang et al., 2008; Alam et al., 2014; Alam and Akbar, 2013; Alam et al., 2013; Bekir, 2008; Roshid et al., 2013a,b; Neyrame et al., 2012; Akbar et al., 2012), the F-expansion method (Wang and Zhou, 2003; Wang and Zhou, 2005), the Backlund transformation method (Miura, 1978), the Darboux transformation method (Matveev and Salle, 1991), the Homogeneous balance method (Wang, 1995, 1996; Zayed et al., 2004), the Adomian decomposition method (Adomain, 1994; Wazwaz, 2002), the Auxiliary equation method (Sirendaoreji, 2003, 2007), the Simplest equation (Eslami et al., 2013; Yildirim et al., 2012) and so on.

Recently, Wang and Xu (2013a,b) and Wang et al. (2014), presented some exact solutions of different nonlinear evolution equations by Lie group analysis. Wang and Xu (2013a) established exact solutions of nonlinear time fractional Sharma-Tasso-Olver equation via Lie group analysis. Zhao and Li (2013) proposed the $\exp(-\varphi(\eta))$ -expansion method to find new type of solutions for nonlinear evolution equations. In this work, we apply exp $(-\phi(\xi))$-expansion method to solve the Shorma-Tasso-Olver equation.

*Corresponding author. E-mail: harunorroshidmd@gmail.com

Mathematics Subject Classification: 35C07, 35C08, 35P99.

METHODOLOGY

In this section we describe exp $(-\phi(\xi))$- expansion method for finding traveling wave solutions of nonlinear evolution equations (Miura, 1978). Suppose that a nonlinear equation, say in two independent variables x and t is given by:

$$F(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \ldots\ldots\ldots\ldots) = 0 \quad (1)$$

where $u(\xi) = u(x,t)$ is an unknown function, F is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method:

Step 1

Combining the independent variables *x* and *t* into one variable $\xi = x \pm \omega t$, we suppose that

$$u(x,t) = u(\xi),\ \xi = x \pm \omega t \quad (2)$$

The travelling wave transformation Equation (2) permits us to reduce Equation (1) to the following ordinary differential equation (ODE):

$$\Re(u, u', u'', \ldots\ldots\ldots) = 0 \quad (3)$$

where $\Re$ is a polynomial in $u(\xi)$ and its derivatives, whereas $u'(\xi) = \frac{du}{d\xi}$, $u''(\xi) = \frac{du^2}{d\xi^2}$ and so on.

Step 2

We suppose that Equation (3) has the formal solution:

$$u(\xi) = \sum_{i=0}^{n} A_i (\exp(-\phi(\xi)))^i \quad (4)$$

Where $A_i (0 \le i \le n)$ are constants to be determined, such that $A_n \ne 0$ and $\phi = \phi(\xi)$ satisfies the following ODE:

$$\phi'(\xi) = \exp(-\phi(\xi)) + \mu \exp(\phi(\xi)) + \lambda \quad (5)$$

Equation (5) gives the following solutions:

When $\lambda^2 - 4\mu > 0, \mu \ne 0$,

$$\phi(\xi) = \ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\xi + E)\right) - \lambda}{2\mu}\right) \quad (6)$$

When $\lambda^2 - 4\mu < 0, \mu \ne 0$,

$$\phi(\xi) = \ln\left(\frac{\sqrt{(4\mu - \lambda^2)}\tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\xi + E)\right) - \lambda}{2\mu}\right) \quad (7)$$

When $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \ne 0$,

$$\phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right) \quad (8)$$

When $\lambda^2 - 4\mu = 0, \mu \ne 0, \lambda \ne 0$,

$$\phi(\xi) = \ln\left(-\frac{2(\lambda(\xi + E) + 2)}{\lambda^2(\xi + E) - 1}\right), \quad (9)$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$, $\phi(\xi) = \ln(\xi + E)$ (10)

$A_n, \ldots\ldots\omega, \lambda, \mu$ are constants to be determined later, $A_n \ne 0$, the positive integer *n* can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Equation (3).

Step 3

We substitute Equation (4) into Equation (3) and then we account the function $\exp(-\phi(\xi))$. As a result of this substitution, we get a polynomial of $\exp(-\phi(\xi))$. We equate all the coefficients of same power of $\exp(-\phi(\xi))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_N, \cdots\cdots, V, \lambda, \mu$. Substituting the values of $A_N, \cdots\cdots, V, \lambda, \mu$ into Equation (4) along with general solutions of Equation (5) completes the determination of the solution of Equation (1).

Application of the method

Here we will present the $\exp(-\phi(\xi))$ expansion method to construct the exact solutions and then the solitary wave solutions of the Shorma-Tasso-Olver equation. First consider the Shorma-Tasso-Olver equation in the forms:

$$u_t + a(u^3)_x + \frac{3}{2}a(u^2)_{xx} + au_{xxx} = 0 \tag{11}$$

Using the wave transformation $u(x,t) = u(\xi), \xi = x - vt$ Equation (12) is carried to an ODE

$$-vu' + a(u^3)' + \frac{3}{2}a(u^2)'' + au''' = 0 \tag{12}$$

Equation (12) is integrable, therefore, integrating with respect to ξ once yields:

$$C - vu + au^3 + \frac{3}{2}a(u^2)' + au'' = 0 \tag{13}$$

Now, balancing the highest order derivative u'' and non-linear term u^3, we get $n = 1$. Therefore, the solution of Equation (13) is of the form:

$$u(\xi) = A_0 + A_1(\exp(-\Phi(\xi))), \tag{14}$$

where A_0, A_1 are constants to be determined such that $A_N \neq 0,$ while λ, μ are arbitrary constants.

Substituting Equation (14) into Equation (13) and then equating the coefficients of $\exp(-\phi(\xi))$ to zero, we get:

$$-3aA_1A_0\mu - VA_0 + aA\mu\lambda + aA_0^3 + P = 0 \tag{15}$$

$$2aA_1\mu + 3aA_0^2A_1 - 3aA_0A_1\lambda - VA_1 - 3aA_1^2\mu + aA_1\lambda = 0 \tag{16}$$

$$3aA_1\lambda - 3aA_0A_1 - 3aA_1A_0 - 3aA_1^2\lambda + 3aA_0A_1^2 = 0 \tag{17}$$

$$2aA_1 + aA_1^3 - 3aA_1^2 = 0 \tag{18}$$

Solving the Equation (15) - Equation (18), yields:

Case 1

$$P = 2aA_0\mu + 2aA_0^3 - 3aA_0^2\lambda + aA_0\lambda^2 - a\mu\lambda$$

$$V = -a\mu + 3aA_0^2 - 3aA_0\lambda + a\lambda^2, A_0 = A_0, A = 1$$

Case 2

$$V = -4a\mu + a\lambda^2, A_0 = \lambda, A_1 = 2$$

For Case 1:

Now substituting the values of V, A_0, A_1 into Equation (14) yields

$$u(\xi) = A_0 + \exp(-\phi(\xi)), \tag{19}$$

where $\xi = x + a(\mu - 3A_0^2 + 3A_0\lambda - \lambda^2)t$

Now substituting Equation (6) - Equation (10) into Equation (19) respectively, we get the following five traveling wave solutions of modified equal width equation.

When $\mu \neq 0, \ \lambda^2 - 4\mu > 0,$

$$u_1(\xi) = A_0 - \left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + E)) + \lambda}\right)$$

where $\xi = x + a(\mu - 3A_0^2 + 3A_0\lambda - \lambda^2)t$, E is an arbitrary constant.

When $\mu \neq 0, \ \lambda^2 - 4\mu < 0,$

$$u_2(\xi) = A_0 + \left(\frac{2\mu}{\sqrt{(4\mu - \lambda^2)}\tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\xi + E)\right) - \lambda}\right)$$

where $\xi = x + a(\mu - 3A_0^2 + 3A_0\lambda - \lambda^2)t$, E is an arbitrary constant.

When $\mu = 0, \ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0,$

$$u_3(\xi) = A_0 + \frac{\lambda}{\exp(\lambda(\xi + E)) - 1}$$

where $\xi = x + a(-3A_0^2 + 3A_0\lambda - \lambda^2)t$, E is an arbitrary constant.

When $\mu \neq 0, \ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$u_4(\xi) = A_0 - \frac{\lambda^2(\xi + E) - 1}{2(\lambda(\xi + E) + 2)}$$

where $\xi = x + a(\mu - 3A_0^2 + 3A_0\lambda - \lambda^2)t$, E is an arbitrary constant.

When $\mu = 0, \ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$ $u_5(\xi) = A_0 + \frac{1}{\xi + E}$

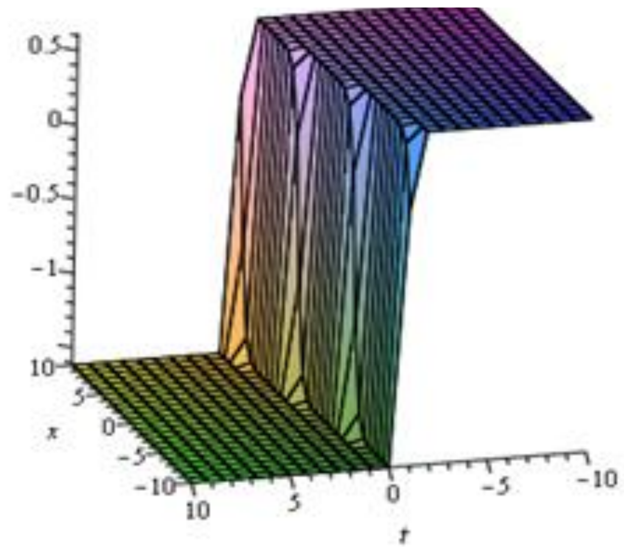

Figure 1. Kink solution of u_1 with $A_0 = 1, A_1 = 1, a = 4, \lambda = 3, \mu = 1, E = 3$ and $-10 \le x, t \le 10$.

where $\xi = x + a(\mu - 3A_0{}^2 + 3A_0\lambda - \lambda^2)t$, E is an arbitrary constant.

For Case 2:

Now substituting the values of V, A_0, A_1 into Equation (14) yields

$$u(\xi) = \lambda + 2\exp(-\phi(\xi)), \tag{20}$$

where $\xi = x + a(\lambda^2 - 4\mu)t$

Now substituting Equation (6) - Equation (10) into Equation (20) respectively, we get the following five traveling wave solutions of modified equal width equation.

When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$u_6(\xi) = \lambda - \left(\frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\,\tanh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + E)) + \lambda} \right)$$

where $\xi = x + a(\lambda^2 - 4\mu)t$, E is an arbitrary constant.

When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$u_7(\xi) = \lambda + \left(\frac{4\mu}{\sqrt{(4\mu - \lambda^2)}\,\tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\xi + E)\right) - \lambda} \right)$$

where $\xi = x + a(\lambda^2 - 4\mu)t$, E is an arbitrary constant

When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0,$

$$u_8(\xi) = \lambda\left(1 + \frac{2}{\exp(\lambda(\xi + E)) - 1}\right)$$

where $\xi = x + a\lambda^2 t$, E is an arbitrary constant

When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$u_9(\xi) = \lambda - \frac{\lambda^2(\xi + E) - 1}{(\lambda(\xi + E) + 2)}$$

where $\xi = x$, E is an arbitrary constant

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$ $u_{10}(\xi) = \lambda + \frac{2}{\xi + E}$

where $\xi = x$, E is an arbitrary constant.

GRAPHICAL REPRESENTATION

The graphical demonstrations of obtained solutions for particular values of the arbitrary constants are shown in Figure 1 to 5 with the aid of commercial software Maple 13.

Conclusion

In this paper, we have applied the $\exp(-\phi(\xi))$-expansion method for the exact solution of the Shorma-Tasso-Olver equation and constructed some new exact travelling wave solutions. The travelling wave solutions are expressed by the hyperbolic functions, the trigonometric functions solutions and the rational functions. This paper shows that the $\exp(-\Phi(\eta))$-expansion method is quite efficient and effective to find

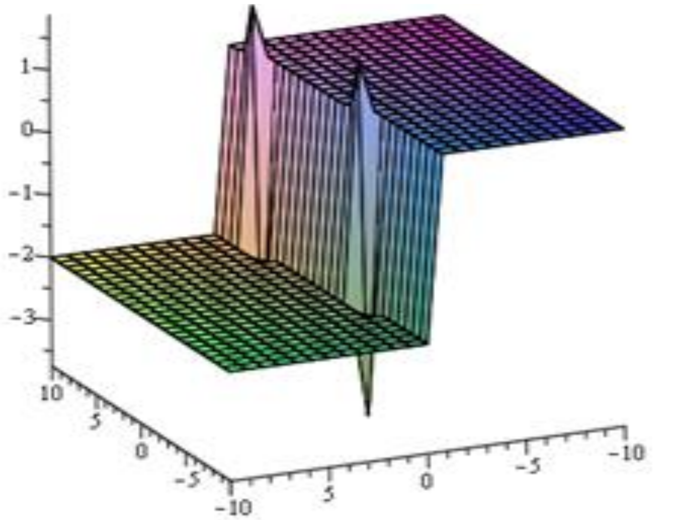

Figure 2. Singular Kink of u_3 with $A_0 = 1, A_1 = 1, a = 4, \lambda = 3, \mu = 1, E = 3$ and $-10 \le x, t \le 10.$

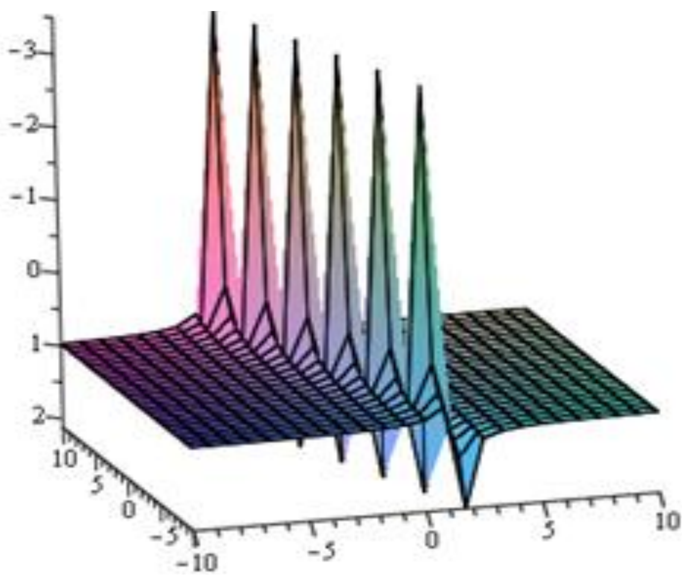

Figure 3. Singular soliton of u_4 when $A_0 = 1, A_1 = 1, a = 4, \lambda = 3, \mu = 1, E = 3$ and $-10 \le x, t \le 10.$

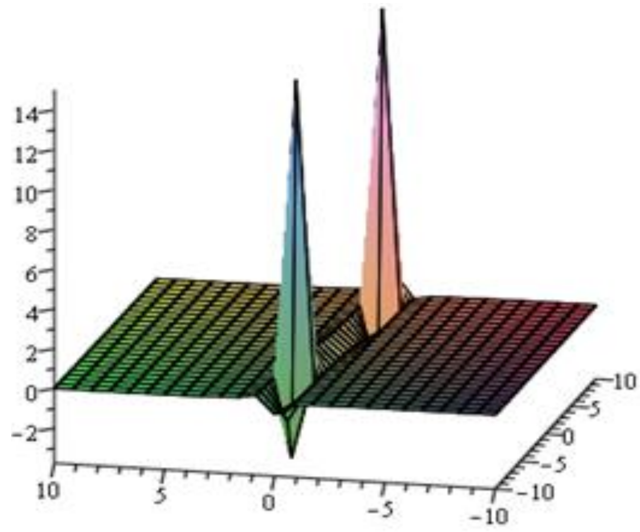

Figure 4. Singular kink of $u_9(\xi)$ when $A_0 = \lambda, A_1 = 2, a = 4, \lambda = 2, \mu = 2, E = 5$ and $-10 \le x, t \le 10.$

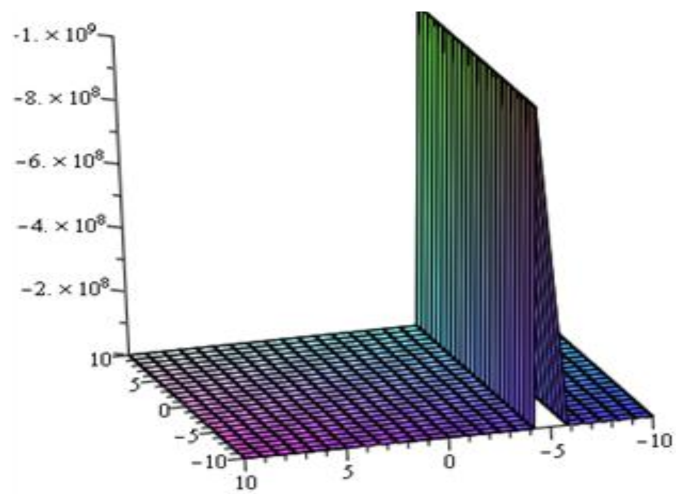

Figure 5. Singular soliton of $u_{10}(\xi)$ when $A_0 = \lambda, A_1 = 2, a = 4, \lambda = 0, \mu = 0, E = 5$ and $-10 \le x, t \le 10.$

the exact solutions of NLEEs. Also, we observe that this method can be also applied to other nonlinear evolution equations.

REFERENCES

Adomain G (1994). Solving frontier problems of physics: The decomposition method, Kluwer Academic Publishers, Boston.

Akbar MA, Ali NHM, Zayed EME (2012). A generalized and improved (G'/G)-expansion method for nonlinear evolution equations, Math. Prob. Eng, P. 22. doi: 10.1155/2012/459879.

Alam MN, Akbar MA, Mohyud-Din ST (2014). A novel (G'/G)-expansion method and its application to the Boussinesq equation, Chin. Phys. B, Article ID 131006. 23(2) 02XXXX.

Alam MN, Akbar MA (2013). Exact traveling wave solutions of the KP-BBM equation by using the new generalized (G'/G)-expansion method, SpringerPlus, 2(1):617. DOI: 10.1186/2193-1801-2-617.

Alam MN, Akbar MA, Khan K (2013). Some new exact traveling wave solutions to the (2+1)-dimensional breaking soliton equation, World Appl. Sci. J. 25(3):500-523.

Bekir A (2008). Application of the (G'/G)-expansion method for nonlinear evolution equations, Phys. Lett. A 372 3400-3406.

Chow KW (1995). A class of exact periodic solutions of nonlinear envelope equation, J. Math. Phys. 36:4125-4137.

Eslami M, Mirzazadeh M, Biswas A. Soliton solutions of the resonant nonlinear Schrödinger's equation in optical fibers with time dependent coefficients by simplest equation approach, J. Modern Optics. 60(19):1627-1636.

Hu JL (2001). A new method for finding exact traveling wave solutions to nonlinear partial differential equations. Phys. Lett. A, 286:175-179, 2001.

Hu JL (2001). Explicit solutions to three nonlinear physical models. Phys. Lett. A, 287:81-89.

Matveev VB, Salle MA (1991). Darboux transformation and solitons, Springer, Berlin.

Miura MR (1978). Backlund transformation, Springer, Berlin.

Neyrame A, Roozi A, Hosseini SS, Shafiof SM (2012). Exact travelling wave solutions for some nonlinear partial differential equations, journal of King Saud University (Science). 22:275-278.

Roshid HO, Alam MN, Hoque MF, Akbar MA (2013). A new extended (G'/G)-expansion method to find exact traveling wave solutions of nonlinear evolution equations, Mathematics and Statistics, 1(3):162-166.

Roshid HO, Rahman N, Akbar MA (2013). Traveling wave solutions of nonlinear Klein-Gordon equation by extended (G'/G)-expansion method, Annals of Pure and Appl. Math. 3(1):10-16.

Sirendaoreji JS (2003). Auxiliary equation method for solving nonlinear partial differential equations, Phys. Lett. A. 309:387-396.

Sirendaoreji JS (2007). Auxiliary equation method and new solutions of Klein-Gordon equations,Chaos Solitions Fractals 31:943-950.

Wang GW, Xu TZ (2013a). Invariant analysis and exact solutions of nonlinear time fractional Sharma-Tasso-Olver equation by Lie group analysis, Nonlinear Dyn. DOI 10.1007/s11071-013-1150-y.

Wang GW, Xu TZ (2013b). Group Analysis and New Explicit Solutions of Simplified Modified Kawahara Equation with Variable Coefficients. Abstract and Applied Analysis. Article ID 139160, 8 pages. http://dx.doi.org/10.1155/2013/139160.

Wang GW, Xu TZ, Ebadi G, Johnson S, Strong AJ, Biswa A (2013). Singular solitons, shock waves, and other solutions to potential KdV equation, Nonlinear Dynamics, 10.1007/s11071-013-1189-9.

Wang GW, Xu TZ, Johnson S, Biswa A (2014). Solitons and Lie group analysis to an extended quantum Zakharov-Kuznetsov equation, Astrophys. Space Sci. 349(1):317-327.

Wang M (1995). Solitary wave solutions for variant Boussinesq equations, Phy. Lett. A, 199:169-172.

Wang ML (1996). Exact solutions for a compound KdV-Burgers equation, Phys. Lett. A 213:279-287.

Wang ML, Li XZ (2005). Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations. Phys. Lett. A 343:48-54.

Wang ML, Li XZ, Zhang J (2008). The (G'/G)-expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A. 372:417-423.

Wang ML, Zhou YB (2003). The periodic wave solutions for the Klein-Gordon-Schrodinger equations. Phys. Lett. A. 318:84-92.

Wazwaz AM (2002). Partial Differential equations: Method and Applications, Taylor and Francis.

Yildirim A, Paghaleh AS, Mirzazadeh M, Moosaei H, Biswas A (2012). New exact traveling wave solutions for DS-I and DS-II equations, Nonlinear Anal. Modell. Control. 17(3):369-378.

Zayed EME, Abourabia AM, Gepreel KA, Horbaty MM (2006). On the rational solitary wave solutions for the nonlinear HirotaCSatsuma coupled KdV system, Appl. Anal. 85:751-768.

Zayed EME, Zedan HA, Gepreel KA (2004). On the solitary wave solutions for nonlinear Hirota-Sasuma coupled KDV equations, Chaos, Solitons and Fractals. 22:285-303.

Zhao MM, Li C (2013). The $\exp(-\Phi(\eta))$-expansion method applied to nonlinear evolution equations, http://www. Paper. Edu. Cn.