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The Existence and Uniqueness of the Solution of a Diffusive Predator-Prey Model with Omnivory and General Nonlinear Functional Response

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

In this work, we consider a three species modified Lesie-Gower food web model with general nonlinear functional response and omnivory which is defined as feeding on more than one trophic level. The carrying capacity of the model is proportional to the population size of the biotic resource plus a const. The main objective of this paper is to investigate the existence and uniqueness of the solution of this model. It is shown that the omnivory has important influence on the existence and uniqueness of the solution of the model.

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1 Introduction

In recent years, one of dominant themes in both ecology and mathematical ecology is the dynamic relationship between predators and their prey due to its universal existence and importance in population dynamics. The investigations on predator-prey models are developed during these thirty years, and more realistic models are derived in view of laboratory experiments and observations (see [1-9]). Since 1970s, there have been some interesting and impressive results on investigating the dynamics of three species predator-prey systems[10-14]. For example, Safuan [14] studied the following Lesie-Gower predator-prey model with the same biotic resource

$$\begin{cases} \dfrac{dx}{dt} = r_1 x(1 - \dfrac{x}{pz}) - axy, \\ \dfrac{dy}{dt} = r_2 y(1 - \dfrac{y}{qz}) + bxy, \\ \dfrac{dz}{dt} = z(c - dx - ey), \end{cases} \tag{1.1}$$

where functions $x(t), y(t), z(t)$ are populations of prey, predator and biotic resource, respectively; and $r_1, r_2, a, b, c, d, e, p, q$ are positive constants. For more biological background of system (1.1), one could refer to [14] and the references cited therein.

On the other hand, we notice that the carrying capacities of both the prey and predator depend on the amount of biotic resource in the above model,that is the carrying capacity of the prey and predator is proportional to the population size of the biotic resource. However, it has somewhat singular behavior at low densities, and thus the model cannot be linearized at the boundary equilibria. Therefore, the linear stability of boundary equilibria are not able to be studied. Indeed, this singularity causes much difficulty in the analysis of the system, contributes significantly to the richness of dynamics of the model. In fact, if this favorite food z is lacking severely, the prey x and predator y will switch to other population, but its growth will be limited. By adding a positive constant to the carrying capacity, the model (1.1) becomes the following modified Lesie-Gower predator-prey model with omnivory

$$\begin{cases} \dfrac{dx}{dt} = r_1 x(1 - \dfrac{x}{k_1 + pz}) - axy, \\ \dfrac{dy}{dt} = r_2 y(1 - \dfrac{y}{k_2 + qz}) + bxy, \\ \dfrac{dz}{dt} = z(c - dx - ey). \end{cases} \tag{1.2}$$

In the evolutionary process of the species, the individuals do not remain fixed in space, and their spatial distribution changes continuously due to the impact of many reasons. Therefore, the spatial component of ecological interactions has been recognized as an important factor. In recent years, different spatial effects have been introduced into population models. Considering the natural diffusion and inhibitory effect, many researchers extend the predator-prey model of ODE to the corresponding diffusive predator-prey model by incorporating the diffusion terms(see[15-36]).

In [31], by adding the diffusion into the model (1.1), Jau investigated the following nonlinear diffusive Lesie-Gower predator-prey model

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \triangle u = r_1 u(1 - \dfrac{u}{pw}) - auv, & x \in \Omega, t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \triangle v = r_2 v(1 - \dfrac{v}{qw}) + buv, & x \in \Omega, t > 0, \\ \dfrac{\partial w}{\partial t} = w(c - du - ev), & x \in \Omega, t > 0, \\ \dfrac{\partial u(t,x)}{\partial \nu} = \dfrac{\partial v(t,x)}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ u(0,x) = u_0(x), v(0,x) = v_0(x), w(0,x) = w_0(x), & x \in \Omega, \end{cases} \tag{1.3}$$

where $u(t,x), v(t,x)$ and $w(t,x)$ are the density of the densities of prey, predator and biotic resource at time t and location $x \in \Omega$, respectively. Ω is a bounded open set in R^n, $\partial\Omega$ is C^1-class, and $u_0(x), v_0(x), w_0(x)$ are Holder continuous functions on Ω. ν is the outward unit normal vector of the boundary $\partial\Omega$. The homogeneous Neumann boundary conditions indicate that the predator-prey system is self-contained with zero population flux across the boundary. He investigated the existence and uniqueness of solution for the system (1.3), that is, he obtained the following result[31].

Theorem A Suppose that constants $\varepsilon, \alpha, \beta, M, N$ and K satisfy

$$\begin{aligned} &0 < \varepsilon \le \min_{x\in\Omega} w_0(x), \alpha \ge \|w_0\|_\infty, \beta \ge c, M \ge \max\{\|u_0\|_\infty, p\alpha e^{\beta T}\}, \\ &N \ge \max\{\|v_0\|_\infty, \frac{r_2 + bM}{r_2} q\alpha e^{\beta T}, \frac{1}{e}(c - dM)\}, K \ge dM + eN - c, \end{aligned} \tag{1.4}$$

then the system (1.3) has a unique solution (u, v, w) on $[0, T] \times \bar{\Omega}$, and

$$(0, 0, \varepsilon e^{-Kt}) \le (u, v, w) \le (M, N, \alpha e^{\beta t}).$$

In this paper, we will focus on three species food web models of predator-prey type with an omnivorous top predator which is defined as feeding on more than one trophic level. Actually, this is a general part of marine or terrestrial food web ecological systems. For one example, species w are plants, species u are herbivores, and species v consume not only plants but also other herbivores. For another example, small vertebrates such as birds and lizards are voracious consumers of both spiders and herbivorous insects. One can find more examples in the complex marine food web systems. This phenomenon has been variously called "trophic level omnivory", "intraguild predation", "higher order predation", or "hyperpredation"[37]. Motivated by the above study, by incorporating the diffusion into the model (1.2), we are interested in studying the following diffusive modified Lesie-Gower predator-prey model with omnivory and general nonlinear functional response

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \triangle u = r_1 u(1 - \dfrac{u}{k_1 + pw}) - \varphi(u)v, & x \in \Omega, t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \triangle v = r_2 v(1 - \dfrac{v}{k_2 + qw}) + b\varphi(u)v, & x \in \Omega, t > 0, \\ \dfrac{\partial w}{\partial t} = w(c - du - ev), & x \in \Omega, t > 0, \\ \dfrac{\partial u(t,x)}{\partial \nu} = \dfrac{\partial v(t,x)}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ u(0,x) = u_0(x), v(0,x) = v_0(x), w(0,x) = w_0(x), & x \in \Omega, \end{cases} \tag{1.5}$$

where $u(t,x), v(t,x)$ and $w(t,x)$ are the density of the densities of prey, predator and biotic resource at time t and location $x \in \Omega$, respectively. The predator consumes the prey with general nonlinear functional response $\varphi(u)$ and contributes to its growth with rate $b\varphi(u)$. The function $\varphi(u)$ is assumed to satisfy the following assumptions which has been studied in detail by Georgescu and Morosanu in [38].

(G) $\varphi(u)$ of C^1-class is increasing on R_+, $\varphi(0) = 0$, and $0 \le \varphi'(u) \le L$ for $u \in R_+$, where $L \ge 0$.

Note that hypothesis (G) is satisfied if function $\varphi(u)$ represents Holling type II functional response, that is, $\varphi(u) = au/(1 + hu)$, in which a is the search rate of the resource and of the intermediate consumer, and h represents the corresponding clearance rate, that is, search rate multiplied by the (supposedly constant) handling time.

In this paper, by further developing the analysis technique of Jau [31],we will prove the existence and uniqueness of solution for the system (1.5) by the methods of the upper and lower solutions [39] and the semigroup theory [40,41]. The rest of the paper are structured in the following way. In the rest of this section, we will introduce the concept of the upper and lower solutions. In Section 2, under the assumption of the existence of the upper solution $\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v}, \tilde{w})$ and lower solution

$\hat{\mathbf{u}} = (\hat{u}, \hat{v}, \hat{w})$ of the problem (1.5), we will show the existence and uniqueness of solution of the problem (1.5) on the sector $\langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle \equiv \{\mathbf{u} = (u, v, w) \in C(\bar{D}_T) : \hat{\mathbf{u}} \le \mathbf{u} \le \tilde{\mathbf{u}}\}$. In Section 3, we will give a pair of upper and lower solutions of the problem (1.5) on $[0, T] \times \bar{\Omega}$, where T is an arbitrary positive number. Then we prove the existence and uniqueness of the solution (u, v, w) of the problem (1.5) on $[0, T] \times \bar{\Omega}$. In Section 4, conclusions are given in the end of paper.

To simplify the notations of the system (1.5), let $u_1 = u, u_2 = v, u_3 = w, u_{1,0} = u_0, u_{2,0} = v_0, u_{3,0} = w_0, L_1 = d_1\Delta, L_2 = d_2\Delta, L_3 = 0, B = \frac{\partial}{\partial v}$, and

$$\begin{aligned} f_1(u_1, u_2, u_3) &= r_1u_1 - \frac{r_1u_1^2}{k_1 + pu_3} - \varphi(u_1)u_2, \\ f_2(u_1, u_2, u_3) &= r_2u_2 - \frac{r_2u_2^2}{k_2 + qu_3} + b\varphi(u_1)u_2, \\ f_3(u_1, u_2, u_3) &= cu_3 - du_1u_3 - eu_2u_3. \end{aligned} \tag{1.6}$$

Suppose that T is an arbitrary positive number and $D_T = (0, T] \times \Omega, S_T = (0, T] \times \partial\Omega$, then system (1.5) can be written in the following form

$$\begin{aligned} &(u_i)_t - L_iu_i = f_i(u_1, u_2, u_3), in\ D_T, i = 1, 2, 3, \\ &Bu_i(t, x) = 0, on\ S_T, i = 1, 2 \\ &u_i(0, x) = u_{i,0}(x), in\ \Omega, i = 1, 2, 3. \end{aligned} \tag{1.7}$$

Let $J_1 = J_2 = J_3 = \{u : u \ge 0\}$, then

$$\begin{aligned} &\frac{\partial}{\partial u_2}f_1 = -\varphi(u_1) \le 0, \frac{\partial}{\partial u_3}f_1 = \frac{r_1pu_1^2}{(k_1 + pu_3)^2} \ge 0, \\ &\frac{\partial}{\partial u_1}f_2 = b\varphi'(u_1)u_2 \ge 0, \frac{\partial}{\partial u_3}f_2 = \frac{r_2qu_2^2}{(k_2 + qu_3)^2} \ge 0, \\ &\frac{\partial}{\partial u_1}f_3 = -du_3 \le 0, \frac{\partial}{\partial u_2}f_3 = -eu_3 \le 0, \end{aligned} \tag{1.8}$$

for all $(u_1, u_2, u_3) \in J_1 \times J_2 \times J_3$. This implies that for all $(u_1, u_2, u_3) \in J_1 \times J_2 \times J_3$, f_1 is monotone nonincreasing in u_2, and monotone nondecreasing in u_3, f_2 is monotone nondecreasing in u_1, and monotone nondecreasing in u_3, f_3 is monotone nonincreasing in u_1, and monotone nonincreasing in u_2. Let $\mathbf{u} = (u_1, u_2, u_3)$ and $f_i(\mathbf{u}) = f_i(u_i, [\mathbf{u}]_{a_i}, [\mathbf{u}]_{b_i}), i = 1, 2, 3$, where $a_i + b_i = 2$, and f_i is monotone nondecreasing in $[\mathbf{u}]_{a_i}$, and monotone nonincreasing in $[\mathbf{u}]_{b_i}$. Then we have the following definition of coupled upper and lower solutions of the system (1.7) (see[39]).

Definition 1.1. A pair of functions $\tilde{\mathbf{u}} = (\tilde{u_1}, \tilde{u_2}, \tilde{u_3})$, $\hat{\mathbf{u}} = (\hat{u_1}, \hat{u_2}, \hat{u_3})$ are called coupled upper and lower solutions of the system (1.7) if $\tilde{u_1}, \tilde{u_2}, \hat{u_1}, \hat{u_2} \in C(\bar{D}_T) \cap C^{1,2}(D_T)$, $\tilde{u_3}, \hat{u_3} \in C(\bar{D}_T) \cap C^{1,0}(D_T)$, and $\tilde{\mathbf{u}} \ge \hat{\mathbf{u}}$(i.e., $\tilde{u_2} \ge \hat{u_1}$) with $\hat{u}_3(t, x) > 0$ in $D_T = [0, T] \times \bar{\Omega}$, and

$$\begin{aligned} &(\tilde{u}_i)_t - L_i\tilde{u}_i - f_i(\tilde{u}_i, [\tilde{u}]_{a_i}, [\hat{u}]_{b_i}) \ge 0,\ in\ D_T, i = 1, 2, 3, \\ &(\hat{u}_i)_t - L_i\hat{u}_i - f_i(\hat{u}_i, [\hat{u}]_{a_i}, [\tilde{u}]_{b_i}) \le 0,\ in\ D_T, i = 1, 2, 3, \\ &B\tilde{u}_i(t, x) \ge 0 \ge B\hat{u}_i(t, x), on\ S_T, i = 1, 2 \\ &\tilde{u}_i(0, x) \ge u_{i,0}(x) \ge \hat{u}_i(0, x), in\ \Omega, i = 1, 2, 3. \end{aligned} \tag{1.9}$$

For a given pair of coupled upper and lower solutions $\tilde{\mathbf{u}}$, $\hat{\mathbf{u}}$, we define the sector $\langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle \equiv \{\mathbf{u} = (u, v, w) \in C(\bar{D}_T) : \hat{\mathbf{u}} \le \mathbf{u} \le \tilde{\mathbf{u}}\}$. Let $\underline{c}_1 = \frac{2r_1\tilde{u}_1}{k_1 + p\hat{u}_3} + L\tilde{u}_2 - r_1$, $\underline{c}_2 = \frac{2r_2\tilde{u}_2}{k_2 + q\hat{u}_3} + b\varphi(\hat{u}_1) - r_2$, $\underline{c}_3 = d\tilde{u}_1 + e\tilde{u}_2 - c$. Hence $\underline{c}_i \in C(\bar{D}_T)$ for each $i = 1, 2, 3$, and by the differential mean value theorem, we can easily obtain that for $\hat{u}_1 \le v_1 \le u_1 \le \tilde{u}_1$,

$$\begin{aligned} f_1(u_1, u_2, u_3) - f_1(v_1, u_2, u_3) &= r_1(u_1 - v_1) - \tfrac{r_1}{k_1 + pu_3}(u_1 + v_1)(u_1 - v_1) \\ &\quad -(\varphi(\hat{u}_1) - \varphi(\hat{v}_1))u_2 \\ &= r_1(u_1 - v_1) - \tfrac{r_1}{k_1 + pu_3}(u_1 + v_1)(u_1 - v_1) \\ &\quad -\varphi'(\xi)(u_1 - v_1)u_2 \\ &\ge (r_1 - \tfrac{2r_1\tilde{u}_1}{k_1 + p\hat{u}_3} - L\tilde{u}_2)(u_1 - v_1) \\ &= -\underline{c}_1(u_1 - v_1), \end{aligned}$$

where $\xi \in (v_1, u_1)$.

Similarly, we can obtain that for $\hat{u}_2 \le v_2 \le u_2 \le \tilde{u}_2$,

$$\begin{aligned} f_2(u_1,u_2,u_3) - f_2(u_1,v_2,u_3) \quad & = r_2(u_2 - v_2) - \tfrac{r_2}{k_2+qu_3}(u_2+v_2)(u_2-v_2) \\ & +b\varphi(u_1)(u_2 - v_2) \\ & \ge (r_2 - \tfrac{2r_2\tilde{u}_2}{k_2+q\tilde{u}_3} + b\varphi(\hat{u}_1))(u_2 - v_2) \\ & = -\underline{c}_2(u_2 - v_2), \end{aligned}$$

and for $\hat{u}_3 \le v_3 \le u_3 \le \tilde{u}_3$,

$$f_3(u_1,u_2,u_3) - f_3(u_1,u_2,v_3) \ge -\underline{c}_3(u_3 - v_3).$$

Thus we prove that there are $\underline{c}_i \in C(\bar{D}_T)$, such that for $\hat{u}_i \le v_i \le u_i \le \tilde{u}_i$,

$$f_i(u_i, [u]_{a_i}, [u]_{b_i}) - f_i(v_i, [u]_{a_i}, [u]_{b_i}) \ge -\underline{c}_i(u_i - v_i), i = 1,2,3. \tag{1.10}$$

Let $\bar{c}_1 = r_1 - \frac{2r_1\hat{u}_1}{k_1+p\tilde{u}_3}$, $\bar{c}_2 = r_2 - \frac{2r_2\hat{u}_2}{k_2+q\tilde{u}_3} + b\varphi(\tilde{u}_1)$, $\bar{c}_3 = c - d\hat{u}_1 - e\hat{u}_2$. Hence $\bar{c}_i \in C(\bar{D}_T)$ for each $i = 1,2,3$, and by the differential mean value theorem, we can easily obtain that for $\hat{u}_1 \le v_1 \le u_1 \le \tilde{u}_1$,

$$\begin{aligned} f_1(u_1,u_2,u_3) - f_1(v_1,u_2,u_3) \quad & = r_1(u_1 - v_1) - \tfrac{r_1}{k_1+pu_3}(u_1+v_1)(u_1-v_1) \\ & -(\varphi(\hat{u}_1) - \varphi(\hat{v}_1))u_2 \\ & = r_1(u_1 - v_1) - \tfrac{r_1}{k_1+pu_3}(u_1+v_1)(u_1-v_1) \\ & -\varphi'(\xi)(u_1 - v_1)u_2 \\ & \le (r_1 - \tfrac{2r_1\hat{u}_1}{k_1+p\tilde{u}_3})(u_1 - v_1) \\ & = \bar{c}_1(u_1 - v_1), \end{aligned}$$

where $\xi \in (v_1, u_1)$.

Similarly, we can obtain that for $\hat{u}_2 \le v_2 \le u_2 \le \tilde{u}_2$,

$$\begin{aligned} f_2(u_1,u_2,u_3) - f_2(u_1,v_2,u_3) \quad & = r_2(u_2 - v_2) - \tfrac{r_2}{k_2+qu_3}(u_2+v_2)(u_2-v_2) \\ & +b\varphi(u_1)(u_2 - v_2) \\ & \le (r_2 - \tfrac{2r_2\hat{u}_2}{k_2+q\tilde{u}_3} + b\varphi(\tilde{u}_1))(u_2 - v_2) \\ & = \bar{c}_2(u_2 - v_2), \end{aligned}$$

and for $\hat{u}_3 \le v_3 \le u_3 \le \tilde{u}_3$,

$$f_3(u_1,u_2,u_3) - f_3(u_1,u_2,v_3) \le \bar{c}_3(u_3 - v_3).$$

Thus we prove that there are $\bar{c}_i \in C(\bar{D}_T)$, such that for $\hat{u}_i \le v_i \le u_i \le \tilde{u}_i$,

$$f_i(u_i, [u]_{a_i}, [u]_{b_i}) - f_i(v_i, [u]_{a_i}, [u]_{b_i}) \le \bar{c}_i(u_i - v_i), i = 1,2,3. \tag{1.11}$$

Let $K_{i,i} = |\bar{c}_i| + |\underline{c}_i|, i = 1,2,3$, and $K_{1,2} = \varphi(\tilde{u}_1), K_{1,3} = \frac{r_1p\tilde{u}_1^2}{(k_1+p\hat{u}_3)^2}, K_{2,1} = bL\tilde{u}_2, K_{2,3} = \frac{r_2q\tilde{u}_2^2}{(k_2+q\hat{u}_3)^2}, K_{3,1} = d\tilde{u}_3, K_{3,2} = e\tilde{u}_3$ on $\bar{D}_T$, and $K_i = K_{i,1} + K_{i,2} + K_{i,3}, i = 1,2,3$. Then $K_{i,j} \in C(\bar{D}_T)$ and $K_i \in C(\bar{D}_T)$ for each $i,j = 1,2,3$, and so $K_{i,j}$ and K_i are bounded functions in $\bar{D}_T$.

It follows from

$$\begin{aligned} & -\varphi(\tilde{u}_1) \le \frac{\partial f_1}{\partial u_2} = -\varphi(u_1) \le 0, \frac{r_1p\tilde{u}_1^2}{(k_1+p\hat{u}_3)^2} \ge \frac{\partial f_1}{\partial u_3} = \frac{r_1pu_1^2}{(k_1+pu_3)^2} \ge 0, \\ & bL\tilde{u}_2 \ge \frac{\partial f_2}{\partial u_1} = b\varphi'(u_1)u_2 \ge 0, \frac{r_2q\tilde{u}_2^2}{(k_2+q\hat{u}_3)^2} \ge \frac{\partial f_2}{\partial u_3} = \frac{r_2qu_2^2}{(k_2+qu_3)^2} \ge 0, \\ & -d\tilde{u}_3 \le \frac{\partial f_3}{\partial u_1} = -du_3 \le 0, -e\tilde{u}_3 \le \frac{\partial f_3}{\partial u_2} = -eu_3 \le 0, \end{aligned}$$

on $\langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$, we can obtain that for all $(u_1, u_2, u_3), (v_1, v_2, v_3) \in \langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$,

$$\begin{aligned} |f_i(u_1, u_2, u_3) - f_i(v_1, v_2, v_3)| &\leq K_{i,1}|u_1 - v_1| + K_{i,2}|u_2 - v_2| + K_{i,3}|u_3 - v_3| \\ &\leq K_i(|u_1 - v_1| + |u_2 - v_2| + |u_3 - v_3|). \end{aligned} \tag{1.12}$$

for each $i = 1, 2, 3$. This inequality shows that f_i satisfies the Lipschitz condition for $\mathbf{u} \in \langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$,$i = 1, 2, 3$. Moreover, f_i is a holder continuous function on $(t, x) \in \bar{D}_T, i = 1, 2, 3$.

Let $F_i(u_i,[\mathbf{u}]_{a_i},[\mathbf{u}]_{b_i}) = f_i(u_i,[\mathbf{u}]_{a_i},[\mathbf{u}]_{b_i}) + \underline{c}_i u_i, i = 1, 2, 3$, then the differential equations in system (1.7) can be written as

$$(u_i)_t - L_i u_i + \underline{c}_i u_i = F_i(u_i, [u]_{a_i}, [u]_{b_i}), in\ D_T, i = 1, 2, 3.$$

From Lemma 8.1 in [39], we can obtain the following lemma.

Lemma 1.2. For each $\mathbf{u} \in \langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$, we denote that $F_i(\mathbf{u})(t, x) = F_i(\mathbf{u}(t, x))$ on $\bar{D}_T, i = 1, 2, 3$. If $\mathbf{u} \in C^{\alpha}(D_T)$, and $\alpha \in (0, 1)$, then the function $F_i(\mathbf{u})$ is Holder continuous in D_T for every $i = 1, 2, 3$. Moreover, if $\mathbf{u}, \mathbf{v} \in \langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$, satisfy that $\mathbf{u} \geq \mathbf{v}$, then

$$F_i(u_i, [u]_{a_i}, [v]_{b_i}) - F_i(v_i, [v]_{a_i}, [u]_{b_i}) \geq 0, i = 1, 2, 3.$$

2 Existence and Uniqueness of the Solution on $\langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$

In this section, we always assume that the upper solution $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$ and lower solution $\hat{\mathbf{u}} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$ of the system (1.7) exist. Let $A_i u_i = (u_i)_t - L_i u_i + \underline{c}_i u_i$ for all $i = 1, 2, 3$. We choose $\bar{\mathbf{u}}^{(0)} = \tilde{\mathbf{u}}$ and $\underline{\mathbf{u}}^{(0)} = \hat{\mathbf{u}}$ as two initial iterations and construct the maximal and minimal sequences $\bar{\mathbf{u}}^{(k)} = (\bar{u}_1^{(k)}, \bar{u}_2^{(k)}, \bar{u}_3^{(k)})$, $\underline{\mathbf{u}}^{(k)} = (\underline{u}_1^{(k)}, \underline{u}_2^{(k)}, \underline{u}_3^{(k)})$, from the iteration process

$$\begin{aligned} A_i \bar{u}_i^{(k)} &= F_i(\bar{u}_i^{(k-1)}, [\bar{u}^{(k-1)}]_{a_i}, [\underline{u}^{(k-1)}]_{b_i})\ in\, D_T, i = 1, 2, 3, \\ A_i \underline{u}_i^{(k)} &= F_i(\underline{u}_i^{(k-1)}, [\underline{u}^{(k-1)}]_{a_i}, [\bar{u}^{(k-1)}]_{b_i})\ in\, D_T, i = 1, 2, 3, \end{aligned}$$

the boundary and initial conditions are given by

$$\begin{aligned} &B\bar{u}_i^{(k)}(t, x) = B\underline{u}_i^{(k)}(t, x) = 0\ on\, S_T, i = 1, 2, \\ &\bar{u}_i^{(k)}(0, x) = \underline{u}_i^{(k)}(0, x)\ in\, \Omega, i = 1, 2, 3. \end{aligned}$$

Before proving the monotone property of the maximal and minimal sequences, we state the following positive lemmas which were given in [39].

Lemma 2.1. Let $u \in C(\bar{D}_T) \cap C^{1,2}(D_T)$ be such that

$$\begin{aligned} &\frac{\partial u}{\partial t} - \alpha \Delta u + \beta u \geq 0,\ for\, all\, 0 < t \leq T, x \in \Omega, \\ &\frac{\partial}{\partial \nu} u(t, x) \geq 0,\ for\, all\, 0 < t \leq T, x \in \partial\Omega, \\ &u(0, x) \geq 0,\ for\, x \in \Omega, \end{aligned}$$

where $\alpha > 0$ and $\beta = \beta(t, x)$ is a bounded function in $D_T = (0, T] \times \Omega$. Then $u(t, x) \geq 0$ in D_T. Moreover $u(t, x) > 0$ in D_T unless it is identically zero in D_T.

Lemma 2.2. Let $u \in C(\bar{D}_T) \cap C^{1,2}(D_T)$ be such that

$$\begin{aligned} &\frac{\partial u}{\partial t} + \beta u \geq 0,\ for\, all\, 0 < t \leq T, x \in \Omega, \\ &u(0, x) \geq 0,\ for\, x \in \Omega, \end{aligned}$$

where $\beta = \beta(t, x)$ is a bounded function in $D_T = (0, T] \times \Omega$. Then $u(t, x) \geq 0$ in D_T. Moreover $u(t, x) > 0$ in D_T unless it is identically zero in D_T.

Now, we will show the monotone property of the maximal and minimal sequences,that is, we can obtain the following Theorem.

Theorem 2.3. Suppose that $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$ and $\hat{\mathbf{u}} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$ are Holder continuous in x, uniformly in D_T, and $u_{1,0}, u_{2,0}$ are Holder continuous on the domain $\bar{\Omega}$ satisfying the boundary condition at $t = 0$ and $u_{3,0}$ is Holder continuous on the domain $\bar{\Omega}$. Then the maximal and minimal sequences $\{\bar{\mathbf{u}}^{(k)}\}$, $\{\underline{\mathbf{u}}^{(k)}\}$ are well-defined on D_T, and they possess the monotone property

$$\hat{u} \leq \underline{u}^{(k)} \leq \underline{u}^{(k+1)} \leq \bar{u}^{(k+1)} \leq \bar{u}^{(k)} \leq \tilde{u}, \tag{2.1}$$

in D_T for every k. Moreover, for each integer k, $\bar{\mathbf{u}}^{(k)}$ and $\underline{\mathbf{u}}^{(k)}$ are coupled upper and lower solutions of the system (1.7).

Proof. The proof of Theorem 2.3 is similar to the proof of Theorem 2.3 in [31], thus it is omitted here.

In view of the monotone property the pointwise (and componentwise) limits

$$\lim_{k\to\infty} \bar{u}^{(k)}(t,x) = \bar{u}^{(k)}(t,x), \lim_{k\to\infty} \underline{u}^{(k)}(t,x) = \underline{u}^{(k)}(t,x) \tag{2.2}$$

exist and satisfy the relation $\hat{\mathbf{u}} \leq \underline{\mathbf{u}} \leq \bar{\mathbf{u}} \leq \tilde{\mathbf{u}}$ in D_T. To show that the system (1.7) has a unique solution in $\langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$, we should prove that $\underline{\mathbf{u}} \leq \bar{\mathbf{u}}$ in D_T.

Theorem 2.4. Let $\tilde{\mathbf{u}}, \hat{\mathbf{u}}$ be coupled upper and lower solutions of the system (1.7). Then there exists a unique solution $\mathbf{u}^*$ to the system (1.7) and $\mathbf{u}^* \in \langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle$. Moreover, the sequences $\bar{\mathbf{u}}^{(k)}, \underline{\mathbf{u}}^{(k)}$ given by the iteration process with initial iterations $\bar{\mathbf{u}}^{(0)} = \tilde{\mathbf{u}}$ and $\underline{\mathbf{u}}^{(0)} = \hat{\mathbf{u}}$ both converge monotonically to $\mathbf{u}^*$.

Proof. The proof of Theorem 2.4 is similar to the proof of Theorem 2.4 in [31], thus it is omitted here.

3 Existence and Uniqueness of Solution of System (1.5)

First, we will prove that the existence of the upper and lower solutions of the system (1.5) on $\bar{D}_T = [0, T] \times \bar{\Omega}$. It is shown that the existence of the solution of the system(1.5) on $\bar{D}_T$. Next, we will prove that the uniqueness of the solution of the system(1.5) on $\bar{D}_T$. Therefore, there exists a unique solution of the system(1.5) on $\bar{D}_T$, where T is an arbitrary positive number.

Theorem 3.1. Suppose that constants α, β, M, N satisfy

$$\begin{aligned} &\alpha \geq \|u_{3,0}\|_\infty, \beta \geq c, M \geq \max\{\|u_{1,0}\|_\infty, k_1 + p\alpha e^{\beta T}\}, \\ &N \geq \max\{\|u_{2,0}\|_\infty, \frac{r_2 + b\varphi(M)}{r_2}(k_2 + q\alpha e^{\beta T})\}, \end{aligned} \tag{3.1}$$

then a pair of functions $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3) = (M, N, \alpha e^{\beta t})$, $(\hat{u}_1, \hat{u}_2, \hat{u}_3) = (0, 0, 0)$ are coupled upper and lower solutions of the system (1.7) on $[0, T] \times \bar{\Omega}$. Moreover, $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$ and $\hat{\mathbf{u}} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$ are Holder continuous in x, uniformly in $\bar{D}_T$.

Proof. Since $\tilde{u}_1 = M, \hat{u}_1 = 0, \tilde{u}_2 = N, \hat{u}_2 = 0, \tilde{u}_3 = \alpha e^{\beta t}, \hat{u}_3 = 0$, we have

$$\begin{aligned} &\hat{u}_1(t,x) = 0 \leq M = \tilde{u}_1(t,x), \\ &\hat{u}_2(t,x) = 0 \leq N = \tilde{u}_2(t,x), \\ &\hat{u}_3(t,x) = 0 \leq \alpha e^{\beta t} = \tilde{u}_3(t,x). \end{aligned}$$

Therefore,

$$(\hat{u}_1, \hat{u}_2, \hat{u}_3) \le (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3).$$

It follows from $M \ge k_1 + p\alpha e^{\beta T}$ that we have

$$\begin{aligned}(\tilde{u}_1)_t - d_1\Delta\tilde{u}_1 - f_1(\tilde{u}_1, \hat{u}_2, \tilde{u}_3) \quad &= -r_1 M + \tfrac{r_1 M^2}{k_1+p\alpha e^{\beta t}} \\ &= r_1 M(-1 + \tfrac{M}{k_1+p\alpha e^{\beta t}}) \\ &\ge r_1 M(-1 + \tfrac{M}{k_1+p\alpha e^{\beta T}}) \\ &\ge 0.\end{aligned}$$

Since $\hat{u}_1 = 0$ and $f_1(u_1, u_2, u_3) = r_1 u_1 - \frac{r_1 u_1^2}{k_1+pu_3} - \varphi(u_1)u_2$, we have

$$(\hat{u}_1)_t - d_1\Delta\hat{u}_1 - f_1(\hat{u}_1, \tilde{u}_2, \hat{u}_3) = 0.$$

It follows from $N \ge \frac{r_2+b\varphi(M)}{r_2}(k_2 + q\alpha e^{\beta T})$ that we have

$$\begin{aligned}(\tilde{u}_2)_t - d_2\Delta\tilde{u}_2 - f_2(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3) \quad &= -r_2 N + \tfrac{r_2 N^2}{k_2+q\alpha e^{\beta t}} - b\varphi(M)N \\ &= N(-r_2 + \tfrac{r_2 N}{k_2+q\alpha e^{\beta t}} - b\varphi(M)) \\ &\ge N(-r_2 + \tfrac{r_2 N}{k_2+q\alpha e^{\beta T}} - b\varphi(M)) \\ &\ge 0.\end{aligned}$$

Since $\hat{u}_2 = 0$ and $f_2(u_1, u_2, u_3) = r_2 u_2 - \frac{r_2 u_2^2}{k_2+qu_3} + b\varphi(u_1)u_2$, we have

$$(\hat{u}_2)_t - d_2\Delta\hat{u}_2 - f_2(\hat{u}_1, \hat{u}_2, \hat{u}_3) = 0.$$

Since $\beta \ge c$ and $f_3(u_1, u_2, u_3) = cu_3 - du_1u_3 - eu_2u_3$, we can easily get that

$$(\tilde{u}_3)_t - f_3(\hat{u}_1, \hat{u}_2, \tilde{u}_3) = \alpha\beta e^{\beta t} - c\alpha e^{\beta t} = (\beta - c)\alpha e^{\beta t} \ge 0.$$

Since $\hat{u}_3 = 0$ and $f_3(u_1, u_2, u_3) = cu_3 - du_1u_3 - eu_2u_3$, we have

$$(\hat{u}_3)_t - f_3(\tilde{u}_1, \tilde{u}_2, \hat{u}_3) = 0.$$

From $\tilde{u}_1 = M, \hat{u}_1 = 0, \tilde{u}_2 = N, \hat{u}_2 = 0$, we obtain that for all $t > 0, x \in \partial\Omega$,

$$\frac{\partial}{\partial\nu}\hat{u}_1(t, x) = \frac{\partial}{\partial\nu}\tilde{u}_1(t, x), \frac{\partial}{\partial\nu}\hat{u}_2(t, x) = \frac{\partial}{\partial\nu}\tilde{u}_2(t, x).$$

It follows from that $M \ge \|u_{1,0}\|_\infty = \max\{u_{1,0}(x), x \in \bar{\Omega}\}, \hat{u}_1 = 0$, we have

$$\hat{u}_1(0, x) = 0 \le u_{1,0}(x) \le \max\{u_{1,0}(x), x \in \bar{\Omega}\} \le M = \tilde{u}_1(0, x).$$

Similarly, from $N \ge \|u_{2,0}\|_\infty = \max\{u_{2,0}(x), x \in \bar{\Omega}\}, \hat{u}_2 = 0$, we have

$$\hat{u}_2(0, x) = 0 \le u_{2,0}(x) \le \max\{u_{2,0}(x), x \in \bar{\Omega}\} \le M = \tilde{u}_2(0, x).$$

From $\alpha \ge \|u_{3,0}\|_\infty = \max\{u_{3,0}(x), x \in \bar{\Omega}\}, \hat{u}_3 = 0$, we can obtain that

$$\hat{u}_3(0, x) = 0 \le u_{3,0}(x) \le \max\{u_{3,0}(x), x \in \bar{\Omega}\} \le \alpha = \tilde{u}_3(0, x).$$

Therefore, a pair of functions $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3) = (M, N, \alpha e^{\beta t})$, $(\hat{u}_1, \hat{u}_2, \hat{u}_3) = (0, 0, 0)$ are coupled upper and lower solutions of the system (1.7) on $[0, T] \times \bar{\Omega}$. The proof is completed.

From Theorem 3.1, we can easily obtain the following result.

Theorem 3.2. Suppose that constants α, β, M, N satisfy

$$\begin{aligned}&\alpha \ge \|w_0\|_\infty, \beta \ge c, M \ge \max\{\|u_0\|_\infty, k_1 + p\alpha e^{\beta T}\}, \\ &N \ge \max\{\|v_0\|_\infty, \frac{r_2 + b\varphi(M)}{r_2}(k_2 + q\alpha e^{\beta T})\},\end{aligned} \tag{3.2}$$

then a pair of functions $(\tilde{u},\tilde{v},\tilde{w}) = (M, N, \alpha e^{\beta t})$, $(\hat{u},\hat{v},\hat{w}) = (0, 0, 0)$ are coupled upper and lower solutions of the system (1.5) on $[0, T] \times \bar{\Omega}$. Moreover, $(\tilde{u},\tilde{v},\tilde{w})$ and $(\hat{u},\hat{v},\hat{w})$ are Holder continuous in x, uniformly in $\bar{D}_T$.

Now, we will prove the limit of the maximal and minimal sequences $\bar{\mathbf{u}}^{(k)} = (\bar{u}_1^{(k)}, \bar{u}_2^{(k)}, \bar{u}_3^{(k)})$, $\underline{\mathbf{u}}^{(k)} = (\underline{u}_1^{(k)}, \underline{u}_2^{(k)}, \underline{u}_3^{(k)})$ with initial iterations $\bar{\mathbf{u}}^{(0)} = (M, N, \alpha e^{\beta t})$ and $\underline{\mathbf{u}}^{(0)} = (0, 0, 0)$ is a unique solution of the system (1.7) on $[0, T] \times \bar{\Omega}$.

Theorem 3.3. Suppose (3.1) holds, then the system (1.7) has a unique solution (u_1, u_2, u_3) on $[0, T] \times \bar{\Omega}$, and

$$(0, 0, 0) \leq (u_1, u_2, u_3) \leq (M, N, \alpha e^{\beta t}).$$

Proof. Suppose that (u_1, u_2, u_3) and (v_1, v_2, v_3) are the solutions of the system (1.7) on $[0, T] \times \bar{\Omega}$, then there is a positive number M_0 such that

$$(0, 0, 0) \leq (u_1, u_2, u_3), (v_1, v_2, v_3) \leq (M_0, M_0, M_0).$$

By similar proof of (1.12) in Section 1, we can easily obtain that there are constants $\tilde{K}_i, i = 1, 2, 3,$ such that

$$|f_i(u_1, u_2, u_3) - f_i(v_1, v_2, v_3)| \leq \tilde{K}_i(|u_1 - v_1| + |u_2 - v_2| + |u_3 - v_3|).$$

Since

$$\begin{aligned}
&(u_1)_t - d_1\Delta u_1 = f_1(u_1, u_2, u_3),\\
&(u_2)_t - d_2\Delta u_2 = f_2(u_1, u_2, u_3),\\
&(u_3)_t = f_3(u_1, u_2, u_3),\\
&(v_1)_t - d_1\Delta v_1 = f_1(v_1, v_2, v_3),\\
&(v_2)_t - d_2\Delta v_2 = f_2(v_1, v_2, v_3),\\
&(v_3)_t = f_3(v_1, v_2, v_3),
\end{aligned}$$

and the boundary and initial conditions

$$\begin{aligned}
&\frac{\partial}{\partial \nu} u_i(t, x) = \frac{\partial}{\partial \nu} v_i(t, x) = 0, x \in \partial\Omega, t > 0, i = 1, 2,\\
&u_i(0, x) = v_i(0, x) = u_{i,0}(x), x \in \Omega, i = 1, 2, 3.
\end{aligned}$$

Let $u_i(t)(x) = u_i(t, x)$ and $v_i(t)(x) = v_i(t, x)$ for all $t \in [0, T]$ and $x \in \bar{\Omega}$. By similar proof of Theorem 2.4 in [31], we can easily obtain that

$$\begin{aligned}
&\|u_1(t) - v_1(t)\| + \|u_2(t) - v_2(t)\| + \|u_3(t) - v_3(t)\|\\
&\leq (\tilde{K}_1 + \tilde{K}_2 + \tilde{K}_3) \int_0^t (\|u_1(s) - v_1(s)\| + \|u_2(s) - v_2(s)\| + \|u_3(s) - v_3(s)\|)ds.
\end{aligned}$$

Hence by Gronwalls inequality, we have $u_i(t, x) = v_i(t, x)$, $i = 1, 2, 3,$ for all $t \in [0, T]$ and $x \in \bar{\Omega}$. From Theorems 2.4 and 3.1, we obtain the system (1.7) has a unique solution (u_1, u_2, u_3) on $[0, T] \times \bar{\Omega}$, and

$$(0, 0, 0) \leq (u_1, u_2, u_3) \leq (M, N, \alpha e^{\beta t}).$$

From Theorem 3.3, we can easily obtain the following result.

Theorem 3.4. Suppose (3.1) holds, then the system (1.5) has a unique solution (u, v, w) on $[0, T] \times \bar{\Omega}$, and

$$(0, 0, 0) \leq (u, v, w) \leq (M, N, \alpha e^{\beta t}).$$

4 Conclusion

In this paper, we consider a three species modified Lesie-Gower food web model with general nonlinear functional response and omnivory which is defined as feeding on more than one trophic level. The carrying capacity of the model is proportional to the population size of the biotic resource plus a const. In [11], Jau also studied a three species Lesie-Gower food web model (1.3)with the same biotic resource, but the carrying capacity of the model is only proportional to the population size of the biotic resource without adding a const. However, it has somewhat singular behavior at low densities, and thus the model cannot be linearized at the boundary equilibria. Indeed, this singularity causes much difficulty in the analysis of the system (1.3). Therefore, the effect of omnivory is considered in the model (1.5). By the methods of the upper and lower solutions and the semigroup theory, we obtain that if (3.1) holds, then the system (1.5) has a unique solution (u, v, w) on $[0, T] \times \bar{\Omega}$, and

$$(0, 0, 0) \leq (u_1, u_2, u_3) \leq (M, N, \alpha e^{\beta t}).$$

In [11], Jau obtained that if (1.4) holds, then the system (1.3) has a unique solution (u, v, w) on $[0, T] \times \bar{\Omega}$, and

$$(0, 0, \varepsilon e^{-Kt}) \leq (u, v, w) \leq (M, N, \alpha e^{\beta t}).$$

Obviously, condition (3.1) is simpler and weaker than condition (1.4). The scope of solutions in our paper is also larger than the scope of solutions in [11]. From above discussion, we can see that the omnivory has important influence on the existence and uniqueness of the solution of the system (1.5). In fact, there are omnivores in Artiodactyla, such as wild boar, which mainly depend on wild fruit, grass, sweet potato, root tuber, tuber and small animals. Since they may eat many kinds of food, in the case of lack of one kind of food such as wild fruit, they can turn to eat another food such as sweet potato, so they can better survive in the vicious environment.

In fact, let $k_i \to 0, i = 1, 2$, and $\varphi(u) = au$, then the system (1.5) can be transformed to the system (1.3). Therefore, we can see more dynamical behaviors of system (1.3) clearly by studying system (1.5). It is shown that our result supplements and complements one of the main results of Jau's paper[Jau GC. The problem of the nonlinear diffusive predator-prey model with the same biotic resource. Nonlinear Anal. Real World Appl. 2017; 34: 188-200].

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Competing Interests

Author has declared that no competing interests exist.

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